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Equivalent value of non-uniform temperature field for thermocouple measurement

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Abstract. The incoming flow is assumed to have a uniform temperature profile when the energy balance of a thermocouple bead is analyzed for the purpose of measurement corrections, which is hardly true in real applications. The incoming temperature is normally non-uniform and the temperature of the flow closer to the bead has a stronger influence on the bead temperature. Considering this, for the first time, a Gaussian probability density function is established to generate an equivalent temperature of the non-uniform temperature field, which can be used in the corrections. With the one-dimensional thermocouple heat transfer program, the bead temperatures are calculated with five non-uniform temperature profiles and compared with the bead temperatures with the uniform temperatures to find the variance correlation of the Gaussian function. The variance is correlated to the Nusselt number and the wire diameter and the bead diameter of the thermocouple. Many cases with different thermocouple sizes, velocities and temperature ranges and widths of the power function profile are simulated. For all the tested cases, the bead temperature differences between the real profiles and the equivalent profiles are less than 27K, demonstrating the good accuracy of the current method.

Keywords: non-uniform temperature; equivalent temperature; probability density function; thermocouple correction.

1. Introduction

The measured temperature of a thermocouple is its bead temperature, which is determined by the energy balance of the bead. Assuming the bead is a regular sphere and the wires are regular cylinders, the bead energy balance schematic is shown in Fig. 1.

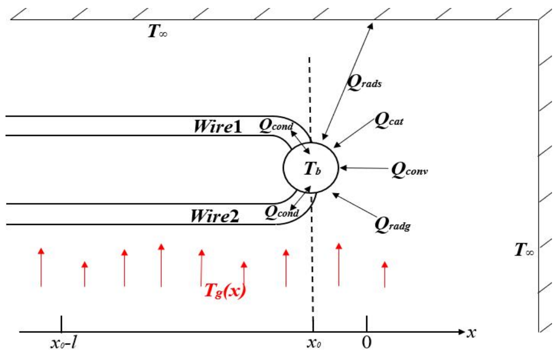

Figure 1 Schematic of bead energy balance

Tg is the gas temperature; T∞ is the environmental temperature; Tb is the bead temperature; Qrads is the radiative heat transfer rate between the bead and the environment; Qconv is the convective heat transfer rate between the bead and the incoming gas; Qradg is the gas radiation absorbed by the bead, which is negligible when the gas is the non-radiative gas or the pressure and/or the size of the radiative gas are small [1]; Qcond is the conduction heat transfer rate between the bead and the wires; Qcat is the catalytic reaction heat release absorbed by the bead, which is negligible for the chemical equilibrium gas [2]. The energy balance of the bead in a steady flow is that the sum of the above terms is zero.

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The temperature of the gas next to bead within certain geometric range influences the wire temperature and the conduction heat transfer rate to the bead, so it influences the bead temperature. The geometric range is quantified by the effective length l ($l=-\ln(0.01)/2\sqrt{kd/h_t}$ for the bare wire thermocouples, where k is the thermal conductivity of the wire with higher conductivity, d is the wire diameter, ht=hw+hr is the total heat transfer coefficient including the convective coefficient hw and the radiative coefficient hr ≈εσ(Tb+T∞)(Tb2+T∞2), σ is the Stefan-Boltzmann constant, and ε is the surface emissivity). Outside the range, the gas temperature has little effect on the bead temperature. Obviously, the temperature of the gas closer to the bead has a stronger effect on the bead temperature and vice versa. It is possible to define a probability density function P(x) (x is the coordinate along the extension direction of the thermocouple as shown in Fig. 1, where the bead coordinate is x0) to measure this kind of weight, so that an equivalent temperature Te of a uniform temperature profile, such as that shown in the following equation, can be searched to reflect the effect of the non-uniform temperature profile Tg(x). The object is that the bead temperature measuring the uniform gas is equal to that measuring the non-uniform gas.

$$T_e=\int_{x_0-l}^{x_0} T_g(x)P(x)dx \tag{1}$$

This kind of equivalence is very common since non-uniform distributions are natural but uniform profiles are normally assumed in the deduction and analysis. For example, Cumpsty [4] has shown that reasonably averaging the non-uniform flow field could simplify the problem, and he has established the equivalent uniform flow field for the non-uniform one. The purpose of the current work is to find a general probability density function to calculate the equivalent temperatures of different non-uniform temperature profiles. The 1D heat transfer program of the bare wire thermocouples [3] is used to calculate the bead temperatures. To validate the accuracy of the method, the cases with different thermocouple sizes and temperature ranges and widths of the non-uniform profiles are simulated.

2. Probability Density Function

Since the gas temperature closer to the bead has a stronger effect on the bead temperature, the normal distribution can be assumed for the probability density function P(x) (from x0-l to x0).

$$\begin{aligned}P(x)&=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-x_0)^2}{2\sigma^2}}\Big/\int_{x_0-l}^{x_0}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-x_0)^2}{2\sigma^2}}dx\\&=\frac{2}{\sqrt{2\pi}\sigma}e^{-\frac{(x-x_0)^2}{2\sigma^2}}\Big/\operatorname{erf}\left(\frac{l}{\sqrt{2}\sigma}\right)\end{aligned} \tag{2}$$

σ is the variance, which is the only unknown variable in the function. Its value will be correlated based on the simulation results.

The wire diameters of thermocouples are normally on the order of 1mm or less and their thermal conductivities are relatively large (30-100W/m/K), so the heat transfer of the thermocouples are mainly in the axial direction and the 1D heat transfer treatment for thermocouples is appropriate. Liu et al. [3] have developed the 1D program for the bare wire thermocouples, which is briefed below. Li et al. [5] have validated the prediction accuracy of the program with the experimental data. For the Hencken flames with the temperature 1184 – 1994 K, the program predicts the bead temperature within the experimental uncertainty, i.e., ±35 K. The schematic of the bead energy conservation is shown in Fig. 2a while that of a wire node is shown in Fig. 2b. The flow is assumed to be the non-radiative gas in equilibrium state, so there is no gas radiation and catalytic reaction heat release. The bead energy conservation (In reality, the two wires are on the same side of the bead as shown in Fig. 1, but the wires are set on different sides of the bead for better illustration):

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$$k_{1.1}\frac{\pi d^2}{4}\left(\frac{T_{1.1}-T_b}{\Delta x}\right)+k_{1.2}\frac{\pi d^2}{4}\left(\frac{T_{2.1}-T_b}{\Delta x}\right)+$$
$$\sigma\varepsilon\left(\pi D^2-\pi d^2/2\right)\left(T_\infty^4-T_b^4\right)+$$
$$h_b\left(\pi D^2-\pi d^2/2\right)\left(T_g(x_0)-T_b\right)=0 \quad (3)$$

The wire node energy conservation:

$$\left(\frac{k_{i+1}+k_i}{2}\right)\frac{\pi d^2}{4}\left(\frac{T_{i+1}-T_i}{\Delta x}\right)+\left(\frac{k_{i-1}+k_i}{2}\right)\frac{\pi d^2}{4}\left(\frac{T_{i-1}-T_i}{\Delta x}\right)$$
$$+\sigma\varepsilon\pi d\Delta x\left(T_\infty^4-T_i^4\right)+h_{w,i}\pi d\Delta x\left(T_g(x_i)-T_i\right)=0 \quad (4)$$

k1.1 and k1.2 are the average thermal conductivities between the first node of wire 1 and the bead and between the first node of wire 2 and the bead. D is the bead diameter. Tb is the bead temperature. Ti and xi are the temperature and the coordinate of the wire node i, respectively. hb and hw,i are the convection coefficients of the bead and the wire node i, respectively, which can be calculated by the following correlations.

Bead in cross flow [6]：

$$Nu_b=2+0.6\,\mathrm{Re}^{1/2}Pr^{1/6} \quad (5)$$

Wire in cross flow [7]：

$$Nu_w=0.42\,\mathrm{Pr}^{1/5}+0.57\,\mathrm{Re}^{1/2}Pr^{1/3} \quad 0.01<\mathrm{Re}<10000 \quad (6)$$

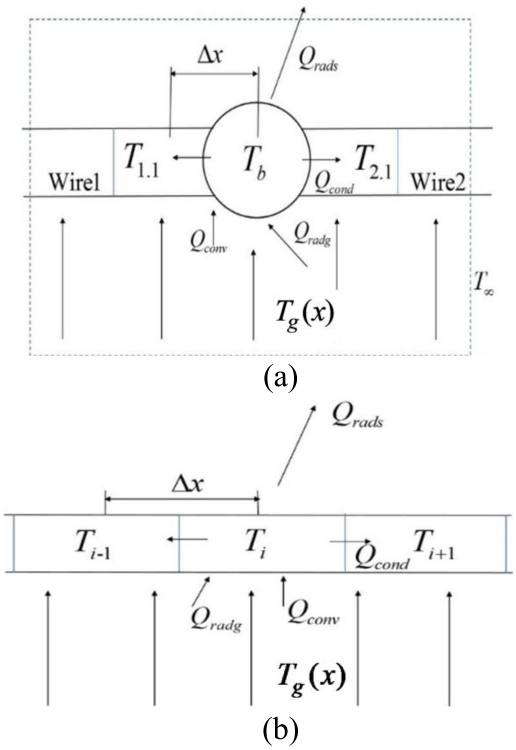

Figure 2 Schematic of bead and wire energy balance discretization: a, thermocouple bead; b, thermocouple wire.

The bare wire S type thermocouple with the bead diameter 0.4mm, the wire diameter 0.125mm, and the wire length 85mm is used as the test thermocouple. One wire of the thermocouple is Pt and the other one is Pt10%Rh, whose thermal conductivities are shown below [8].

$$k_{Pt}=0.0198T+64.141\,;\,k_{Pt-10\%Rh}=0.006T+28.385 \quad (7)$$

where T is the Kelvin temperature. The thermal conductivity of the bead is the average of those of the two wires.

The emissivity of the thermocouple is normally a function of temperature. For the S type thermocouple, the emissivity is assumed to be [9]:

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$$\varepsilon = 0.170 lnT - 0.6395 \quad (8)$$

The flow is nitrogen with 1atm pressure. The calculation of the convection coefficient needs the density, specific heat, conductivity and dynamic viscosity. The density is calculated with the ideal gas law. The specific heat, thermal conductivity and dynamic viscosity are functions of temperature, which are calculated with CHEMKIN formats. Δx=0.05mm, 0.1mm, and 0.2mm are used to test the 1D simulation, the results with Δx =0.1mm and 0.05mm are negligible, so Δx=0.1mm is used in all the simulations. Five incoming temperature profiles are simulated (the bead coordinate x0=-0.01m, the wire coordinate x<x0) as shown in Fig. 3: 1 linear function Tg(x)=169200x+3692; 2 power function 1 Tg(x)=0.007026(-x)-2.727; 3 logarithmic function Tg(x)=-2444ln(-x)-9254; 4 exponential function Tg(x)=13210e188.8x; 5 power function 2 Tg(x)=-3080000(-x)1.83. All the functions give 2000K gas temperature at the bead location. The range of the x coordinate for all the functions is [-0.02m, -0.01m], outside the range, the gas temperature is 300K. The width of the non-uniform profiles is 0.01m, which is more than the effective length. The temperature outside the effective length has no effect on the bead temperature. The incoming velocity are 1-30m/s.

The bead temperature Tb1 is calculated under the non-uniform temperature condition then the effective length l is calculated. The variance σ is tested with (2) to calculate the equivalent temperature Te and the bead temperature Tb2 is calculated with the uniform gas temperature Te. The σ value is adjusted until Tb2=Tb1. The procedure is repeated for all the non-uniform profiles, then the correlation between Nuw and σ/l is searched. Table 1 shows the simulation results.

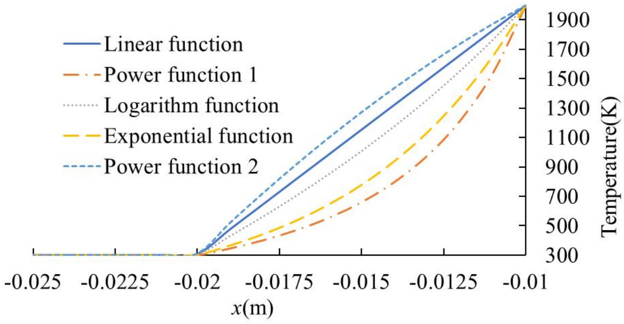

Figure 3 Five incoming gas temperature profiles

Table1. Simulation Result Under Five Different Temperature Profiles Linear Function

V(m/s)	Tb(K)	Te(K)	Nuw	l(cm)	σ(cm)	σ/l
1	1504.15	1812.22	0.683	0.940	0.139	0.147
3	1561.44	1828.16	0.977	0.798	0.127	0.158
5	1589.33	1837.59	1.171	0.739	0.120	0.162
8	1615.72	1847.09	1.373	0.687	0.113	0.164
11	1633.99	1853.81	1.531	0.654	0.108	0.165
15	1652.10	1860.60	1.703	0.623	0.103	0.166
20	1669.15	1866.80	1.876	0.596	0.0984	0.165
30	1693.59	1876.14	2.151	0.559	0.0915	0.164

Exponential Function

V(m/s)	Tb(K)	Te(K)	Nuw	l(cm)	σ(cm)	σ/l
1	1417.20	1659.91	0.713	0.952	0.131	0.137
3	1472.86	1684.41	1.032	0.805	0.120	0.149
5	1501.17	1698.12	1.224	0.744	0.114	0.154
8	1528.69	1712.01	1.432	0.691	0.108	0.156
11	1548.14	1723.78	1.593	0.657	0.103	0.157
15	1567.74	1734.33	1.765	0.626	0.0986	0.158
20	1586.50	1745.38	1.942	0.598	0.0940	0.157
30	1613.86	1762.01	2.222	0.560	0.0872	0.156

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Power Function 1

V(m/s)	Tb(K)	Te(K)	Nuw	l(cm)	σ(cm)	σ/l
1	1367.43	1580.00	0.731	0.958	0.125	0.131
3	1420.85	1603.41	1.058	0.809	0.116	0.143
5	1448.84	1619.68	1.254	0.747	0.110	0.148
8	1476.53	1636.31	1.466	0.693	0.104	0.150
11	1496.40	1648.76	1.629	0.659	0.0996	0.151
15	1516.63	1661.86	1.805	0.627	0.0951	0.152
20	1536.18	1675.34	1.984	0.598	0.0905	0.151
30	1564.99	1695.36	2.267	0.561	0.0839	0.150

Logarithm Function

V(m/s)	Tb(K)	Te(K)	Nuw	l(cm)	σ(cm)	σ/l
1	1473.22	1758.84	0.693	0.945	0.134	0.142
3	1529.85	1777.12	1.005	0.801	0.123	0.154
5	1557.84	1787.24	1.193	0.741	0.117	0.158
8	1584.57	1799.15	1.397	0.689	0.110	0.160
11	1603.23	1805.98	1.554	0.655	0.106	0.161
15	1621.83	1814.67	1.725	0.624	0.101	0.162
20	1639.45	1823.15	1.899	0.596	0.0961	0.161
30	1664.88	1835.05	2.175	0.559	0.0893	0.160

Power Function 2

V(m/s)	Tb(K)	Te(K)	Nuw	l(cm)	σ(cm)	σ/l
1	1523.44	1850.93	0.676	0.937	0.143	0.153
3	1581.47	1866.31	0.980	0.797	0.129	0.162
5	1608.94	1872.85	1.165	0.738	0.123	0.166
8	1635.58	1881.59	1.365	0.686	0.115	0.168
11	1653.58	1886.99	1.520	0.653	0.110	0.168
15	1671.47	1892.41	1.688	0.623	0.105	0.169
20	1687.82	1896.68	1.861	0.595	0.101	0.169
30	1711.95	1905.24	2.133	0.558	0.0930	0.167

Although the temperature range is the same for the five profiles, different distributions give out different bead temperatures and corresponding equivalent temperatures. For example, when the flow velocity is 11m/s, the bead temperature with power function 1 is the lowest 1496.4K and the equivalent temperature is also the lowest 1648.76K. The bead temperature with power function 2 is the highest 1648.76K and the equivalent temperature is 1653.58K. This is consistent with the gas temperature distribution, power function 2 has the highest gas temperature and power function 1 has the lowest gas temperature. Fig. 4 demonstrates the relationship between σ/l and Nuw. It is clearly seen that σ/l increases first then decreases with Nuw. σ/l also relies on the temperature profiles. However, σ/l does not vary much, which is 0.13-0.167, so it is acceptable to ignore the effect of the profiles and consider only the effect of Nuw. The correlation is shown in (9).

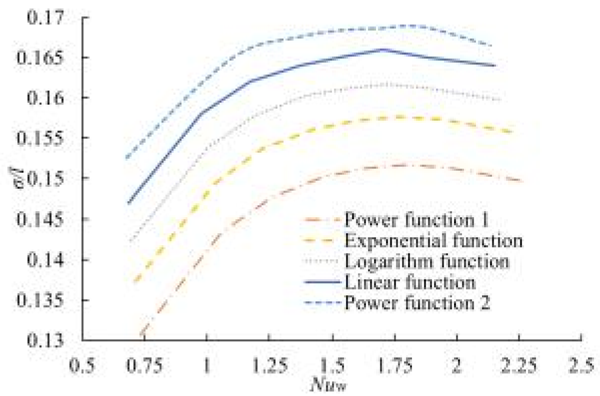

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Figure 4 σ/l-Nuw relationship for five temperature profiles.

$$\sigma / l = 0.0081184 Nu_w^3 - 0.053444 Nu_w^2 + 0.11208 Nu_w + 0.081611 \quad (9)$$

The above correlation was obtained from the simulation result of the thermocouple with the 0.125mm wire diameter and 0.4mm bead diameter. Other thermocouples with different sizes need to be tested. First, the wire diameter is fixed to 0.125mm and the bead diameter is varied within 0.1-1.0mm. The cases with the power function 1 profile and the velocity 3m/s are tested, whose results are shown in table 2. A correction factor Cb=(σ/l)/ (σ/l) D0 is introduced in the table. The correction factor decreases when the bead diameter increases, which is correlated in (10). Keeping D=0.2mm and changing the gas velocity to 1m/s, 11m/s, and 30m/s, the correction factors are 1.27, 1.24, and 1.28, respectively, showing the factor’ s independence on the gas velocity.

Table 2. Simulation Result with Different Bead Diameter while Wire Diameter is Fixed

D(mm)	D/D0(0.4mm)	σ/l	Cb
0.1	0.25	0.197	1.506
0.2	0.50	0.166	1.271
0.3	0.75	0.146	1.121
0.4	1.00	0.131	1.000
0.5	1.25	0.118	0.900
0.6	1.50	0.106	0.814
0.7	1.75	0.0968	0.741
0.8	2.00	0.0884	0.677
0.9	2.25	0.0816	0.625
1.0	2.50	0.0751	0.575

$$C_b = 0.046827\left(\frac{D}{D_0}\right)^4 - 0.3256\left(\frac{D}{D_0}\right)^3 + 0.8976\left(\frac{D}{D_0}\right)^2 - 1.4338\left(\frac{D}{D_0}\right) + 1.8102 \quad (10)$$

The bead diameter is fixed to 0.4mm and the wire diameter is varied within 0.075-0.75mm. The cases with the power function 1 profile and the velocity 3m/s are tested, whose results are shown in table 3. Another correction factor Cw=(σ/l)/ (σ/l) d0 is introduced in the table. The correction factor increases when the wire diameter increases, which is correlated in (11). The simulation results with different gas velocity show the independence of Cw on the velocity.

Table 3.Simulation Result with Different Wire Diameter while Bead Diameter is Fixed

dw(mm)	dw(mm)/(dw0=0.125mm)	σ/l	Cw
0.075	0.6	0.117	0.817
0.125	1.0	0.143	1.000
0.200	1.6	0.164	1.142
0.250	2.0	0.172	1.200
0.300	2.4	0.179	1.245
0.350	2.8	0.183	1.276
0.400	3.2	0.185	1.289
0.500	4.0	0.194	1.355
0.600	4.8	0.202	1.409
0.750	6.0	0.214	1.496

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$$C_w = -0.0035119\left(\frac{d}{d_0}\right)^4 + 0.055492\left(\frac{d}{d_0}\right)^3 - 0.3138\left(\frac{d}{d_0}\right)^2 - 0.8211\left(\frac{d}{d_0}\right) + 0.4306 \tag{11}$$

Combing (9-11) gives the final expression for the variance.

$$\sigma / l = C_b \cdot C_w \cdot (0.0081184 Nu_w^3 - 0.053444 Nu_w^2 + 0.11208 Nu_w + 0.081611), \quad 0.661 \le Nu_w \le 7.545 \tag{12}$$

3. Result and Discussion

Table4 shows the simulated bead temperature comparison between the non-uniform profiles and the equivalent uniform profiles calculated with (12). The thermocouple has the 0.55mm bead diameter and the 0.22mm wire diameter, and the gas velocity is 3m/s. The bead temperature differences are less than 14.31K and the maximum relative error is only 0.87%.

Table 4.Simulated Bead Temperature Comparison for Five Profiles (K)

Function	Linear	Logarithm	Exponential	Power 1	Power 2
Real profile	1602.18	1563.09	1493.03	1432.65	1629.86
Uniform Profile	1616.10	1574.90	1500.32	1428.47	1644.17
Difference	13.92	11.81	7.29	-4.18	14.31
Relative error	0.87%	0.76%	0.49%	-0.29%	0.88%

Four thermocouples with different sizes are simulated and the results are compared: thermocouple 1 d=0.050 mm and D=0.093mm; thermocouple 2 d=0.075 mm and D=0.163mm; thermocouple 3 d=0.125 mm and D=0.399mm; thermocouple 4 d=0.250 mm and D=0.75mm. The profile is power function 1 and the gas velocity is 3m/s (except stated otherwise, this condition applies for the cases in the rest of the paper). Table 5 shows the simulated bead temperature comparison. The relative error is very small and the maximum error is only 1.69%.

Table 5.Simulated Bead Temperature Comparison for Four Thermocouples (K)

Thermocouple	1	2	3	4
Real profile	1596.52	1518.06	1417.79	1246.91
Uniform Profile	1623.47	1528.00	1412.91	1255.79
Difference	26.95	9.94	-4.88	8.88
Relative error	1.69%	0.65%	-0.34%	0.71%

The bead temperatures of thermocouple 3 are simulated when the temperature ranges of power function 1 are adjusted (width is not changed). The results are shown in table 6. The maximum error is 15.47K. The bead temperatures of thermocouple 3 are also simulated when the temperature widths of power function 1 are adjusted (range is not changed). The results are shown in table 7. The maximum error is only 19.93K for both adjustments. When the width is smaller, i.e., the temperature profile is steeper, the error is larger. Fig. 5 shows the bead temperature difference

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variation with the profile width under three different gas velocity (1m/s, 10m/s, and 30m/s). It is clearly seen that the temperature difference is almost not influenced by the velocity.

Table 6. Simulated Bead Temperature Comparison with Adjusted Temperature Range (K)

Range	Real profile	Uniform Profile	Difference
300-1000	817.75	828.90	11.15
300-1600	1198.75	1202.16	3.42
300-2200	1522.85	1507.39	-15.47

Table 7. Simulated Bead Temperature Comparison with Adjusted Temperature Width (K)

Range	Real profile	Uniform Profile	Difference	l/cm
2.2	1457.81	1453.35	-4.47	0.79
1.8	1440.04	1434.97	-5.08	0.79
1.4	1412.88	1406.65	-6.22	0.80
1.0	1420.85	1412.41	-8.44	0.80
0.8	1383.94	1372.14	-11.80	0.80
0.6	1328.15	1308.21	-19.93	0.81

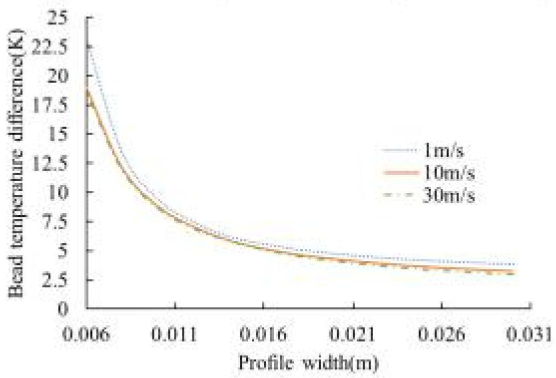

Figure 5 Bead temperature difference variation with profile width under different gas velocity conditions

Conclusion

The gas temperature around the thermocouple wires influences the wire temperature through the convection heat transfer, then the wire temperature influences the conduction rate to the bead and changes the bead temperature. So the gas temperature has effect on the bead temperature. Since the temperature of the gas closer to the bead has more effect on the bead temperature. The normal probability density function is assumed to count this effect with which the non-uniform gas temperature profile can be equivalent to a uniform temperature profile, resulting the same bead temperature.

The cases with five different gas temperature profiles, ten different bead diameters, and ten different bead diameters are simulated with the 1D heat transfer program. The simulation result is used to generate the correlation for the variance of the normal probability density function.

The accuracy of the method is tested by using different sized thermocouples and changing the ranges and widths of the power function profiles. With the wire diameter varying between 0.05mm and 0.25mm and the bead diameter varying between 0.093mm and 0.75mm, the maximum bead temperature difference between the real profiles and the uniform profiles is 26.95K. When the range of the profile is 300-2200K, the bead temperature difference is only 15.47K. When the width of the profile varies between 0.6mm and 2.2mm, the temperature difference only varies between 4.47K

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and 19.93K. The temperature difference is also insensitive to the gas velocity. All these test results have demonstrated the good accuracy of the current method.

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