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Study of predicting honey bee demand based on differential Gaussian model and sensitivity analysis

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Abstract. In recent years, researchers have begun to use CCD to describe this phenomenon of rapidly disappearing bee populations. In this problem, our group will task to integrate and summarize the possible effects of honeybee population changes on farms by modeling changes in the number of bees in an individual hive and simultaneously studying the relationship between bee populations and farmland area.Firstly we use a differential based Gaussian function to build a general model to represent the change in the number of honey bee populations over time. Secondly, we do the sensitivity analysis of the input variables above. Thirdly, the relationship between crop and land area need to be figured out first. We finally find that 13 bee hives are needed.

Keywords: honeybee, modeling. Differential Gaussian model.

1. Background

With the booming of economy and advancing of technology, people have gradually become aware of the importance of protecting the environment. In real-life situations, when a large number of bees disappear, crops and forages will lose their pollen dispersal pathways, and in the long run, the crop and livestock production will be reduced, and eventually there might be a massive food shortage. In summary, a mathematical model to represent the change of population of bee colony over time as a way to conduct sensitivity analysis of its species and to study the relationship between honeybee population and land area can effectively help farmers to keep honeybees more scientifically and rationally and to protect honeybee populations and the local ecological environment.

2. Problem Restatement

Because we are not clear about the reason of CCD problem, in order to investigate the inner link between the population of honeybees and time ,which based on variable factors such as lifespan of honeybees and rate of fertilization ,we will set a population change model of honeybees. In this case, our overall objective is to study the function of change in the size of honeybee colony over time, and to model the dynamic relationship between these changes in the colony. Moreover, we will perform sensitivity analysis on the model to determine the extent to which different factors affect honeybee colony size. Therefore, we will base it on three different dependent variables: forager lifespans, egg laying rates, fertilized and unfertilized egg ratios. On this basis, we will study the changing curve of the honeybee population when the rate of egg production, the proportion of fertilized eggs, and the lifespan change, and discuss the changing pattern of the curve of the population size of the colony.

3. Assumptions

1. 365days in a year.
2. Type of bee is Italian Bee.
3. No external influence (Pesticides, Natural Disasters, Human interference)

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4. All bees die naturally, the queen is always alive, but there is only one, which is negligible compared to the total population of the colony.
5. We use different notations for each of the variables and identify the three main variables of modelling: egg laying rate L(t), fertilization rate f(t) and lifespan L(t). For other possible influences such as environment, food, temperature, climate, etc., we will also take into consideration, but not as variables, instead we will analyze the way they affect our three variables. Our function of the three variables will reflect the variation over 365 days of the year, i.e., in relation to time.
6. Except from the foragers, the lifespans of other bees are the constants because they do not need to expend a lot of energy flying for long periods of time without rest, as this would greatly reduce their life expectancy.
7. Each flower will be visited by γ bee on average.

4. Population Model of bee colony

our population model will base on the following equation:

$$P(t+\Delta t)-P(t)=\big(B(t)-D(t)\big)\cdot t$$

Where P(t) is the population of honeybee colony, and $P(t+\Delta t)-P(t)$ refers the change in population size over a particular time period Δt; B(t) is the birth rate, i.e. the rate of eggs laid in the hive, or the number of eggs laid by the queen per day; D(t) is the death rate, i.e. the death rate of all kinds of bees in the colony expect the queen, which is closely related to the lifespan of honeybees.

In this case, since the birth rate B(t) is influenced by the egg laying rate L(t) and the fertilization rate f(t), the population increase at a particular moment could be shown under the following logical relationships:

$$P(t+\Delta t)-P(t)=P_a+P_b+P_d=(B(t)-D(t))\cdot t$$

Px is the population of species that increased during the time Δt. The overall increase in the population of the bee colony equals to the sum of the increased number of adult bees, brood and drones during that moment.

$$P_d=\big(1-f(t)\big)\cdot L(t)-D_d$$

Moreover, the drone population increase is also equivalent to the number of survived unfertilized eggs minus the death rate of drones. Dx is the death rate for bees.k is the Hive bee: Forager ratio in the colony.

$$P_a+P_b=P_h+P_f+P_b=(1-k)f(t)\cdot L(t)+kf(t)\cdot L(t)+f(t)\cdot L(t)-(D_h+D_f+D_b)$$

$$=2f(t)\cdot L(t)-(D_h+D_f+D_b)$$

By combining the two equations, the sum of change in population is

$$P_a+P_b+P_d=\big(1+f(t)\big)\cdot L(t)-(D_h+D_f+D_b+D_p)$$

$$\begin{cases} D_h=D_b=D(t+21) \\ D_d=D(t+105) \\ D_f=D(t+l(t)) \end{cases}$$

Furthermore, at the current moment is t, the lifespan of a bee born at the current moment is l(t), and the mortality or death rate is D(t + l(t)), where $l(t)$ is the lifespan of the forages at day t. Hance, our basic population model is

$$\therefore P(t+\Delta t)-P(t)=[\big(1+f(t)\big)\cdot L(t)-(D(t+105)+2D(t+21)+D\big(t+l(t)\big)]\cdot t$$

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4.1 Egg laying rate

$$L(t) = N * e^{(-\frac{t-\mu}{2\sigma^2})}$$

In the Gaussian distribution, N represents a stochastic constant, i.e. the number of eggs laid by the queen in a day, μ represents the day of the year with the highest number of eggs laid, and σ is a random constant controlling the range of the interval to make the Gaussian distribution model fit the topic better.

4.2 Fertilization Rate

Moreover, the number of drones in a hive is usually around 2% of the total population and the rest of the compositions in the hive are brood and adult bees. More than 2% rarely happens because the colony itself suppresses the drone and worker bee populations to achieve balance. Therefore, the fertilization rate at a specific moment for the eggs is:

$$f(t) = 98\% \cdot (1 - 5.8\%)$$

4.3 Lifespan

The lifespan of bees changes with time, or flowering period, which is usually happened in spring and summer. The average lifespan of foragers lf in spring and summer is 42 days. However, in autumn and winter, it can reach the maximum lifespan which is 183 days.

4.4 A differential equation-based model of population of bee colony

Initial Population of honeybee colony P(0): 25000. The average population of a mature hive is $\frac{20000+80000}{2} = 50000$, according to the research, 40% to 60% of the bees will follow the newly born queen and leave the original hive and open a new one. Therefore, we take 50% of 50000 as our initial population of a honeybee colony, which is

$$P(0) = 50\% \cdot 50000 = 25000$$

5. Sensitivity analysis

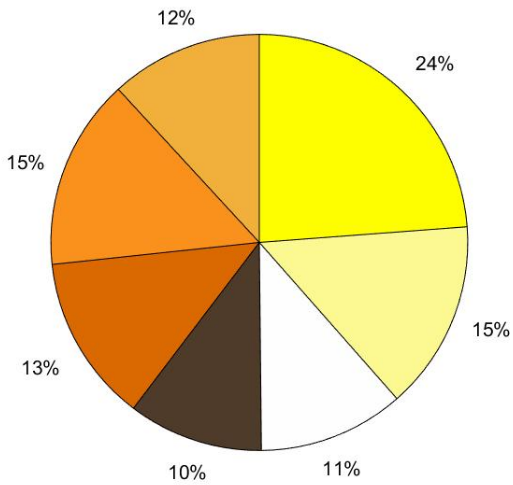

FIG.1 Weighting of each factor

The weight of each factor are shown in the figure above. And among which, the egg laying rate is of greatest importance and the life span is the least important one.

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6. Prediction Model

6.1 Bee population demand measurement model for Crop Pollination

First and foremost, the relationship between crop pollination and bee density have to be shown. The number of bees required to pollinate a unit area of farmland is related to many factors, including the number of flowers, the number of flowers visited by bees (per day), the flowering period, and the number of visits per flower. Therefore, we establish a function of the work task schedule as shown in Eq. (7.1).

$$W = E \times T \times \alpha \tag{7.1}$$

where denotes the workload, that is, the number of flowers that needs to be pollinated per unit area (m2). denotes the work efficiency, that is, the number of flowers a honeybee visits per day. denotes the density of honeybee. This is the number of bees per unit area (m2). denotes the work period, i.e. for a unit area of flowers to be nectar (pollinated) in T days. T corresponds to the period of flower opening.

Workload: the workload of pollinated crops per unit area refers to the number of flowers per square meter and is calculated as follows.

$$W = \text{ Density of crop } \times \text{ numer of flowers per crop} \tag{7.2}$$

Work Efficiency: In section 2, we assume that each flower will be visited by γ bee on average, so the average number of effective visits per bee is: $E = \beta/\gamma$

c) Density of honeybee: It refers to the number of honeybees in unit area.

Therefore, the model in Equation (7.1) can be rewritten as (7.3)

$$\text{Density of crop } \times \text{ numer of flowers per crop} = \beta/\gamma \times T \times \alpha \tag{7.3}$$

From this we can learn that the problem of the demand for bee populations for crop pollination is essentially about getting a certain number of bees per unit area (m2). Equation (7.3) can be rewritten as (7.4):

$$\alpha = \frac{\text{Density of crop } \times \text{ numer of flowers per crop} \times \gamma}{T \times \beta} \tag{7.4}$$

From the above data, we can calculate that α=0.15. Now that we know the number of bees needed per unit area of farmland, the next step we need to determine the distribution of bees thus, finally the total number of bees within that farmland and finally the number of hives H.

6.2 Distribution of honeybee

From assumption, we know that the active range area S is 113040000 m2 (assumed), from which we can find the population size N = 4239000 bees when the bees are uniformly distributed. And since we can know from assumption that there are 3000 bees in a hive, let the number of hives be H. Then H is related to N as a function of (7.5) as follow:

$$H = \frac{N}{3000} \tag{7.5}$$

According to the above data, when we assume a uniform distribution of bees, H = 1413.

But we can first illustrate the probability and density distribution by uniform distribution above because we assume that the probability of bees appearing in any one location is the same, the probability is:

$$N * p(x, y) = \frac{N}{2\pi r^2}$$

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The bee density (average number of bees per square meter) corresponding to 1 square meter is:

$$\frac{N}{2\pi r^2}$$

Considering the effect of density, the bees are unevenly distributed at different distances, and the further the distance, the lower the density. As shown in the figure below, we assume that the bees are evenly distributed over the distance they fly, that is, the probability of a bee appearing at a distance d is

$$p(d)=\frac{1}{r}$$

However, the corresponding range is not as large for different distances. Suppose an individual bee has a flight distance d1 , corresponding to the circle in Fig.1 , and the position corresponding to the distance is the black circle, then the probability of appearing at any point is:

$$\begin{cases} \mathrm{p(x,y)}=\frac{2\mathrm{p(d_2)}}{2\pi \mathrm{d_2}}=\frac{1}{\pi \mathrm{rd_2}} \\ \mathrm{d_2}=\sqrt{\mathrm{x^2+y^2}} \end{cases}$$

Similarly, if its flight distance is d2, corresponding to the circle on the outer green side of Fig.1 , then the probability of appearing at any point is:

$$\begin{cases} \mathrm{p(x,y)}=\frac{2\mathrm{p(d_2)}}{2\pi \mathrm{d_2}}=\frac{1}{\pi \mathrm{rd_2}} \\ \mathrm{d_2}=\sqrt{\mathrm{x^2+y^2}} \end{cases}$$

Comparing the probabilities of the points on the two circles, we can find that the farther the flight distance, the lower the probability. Then adding a bee population of N, the density of its bee distribution for any point (x, y) is:

$$N\times\frac{1}{\pi r d_2}$$

Even further, we assume that the bees are unevenly distributed over distances, i.e., the probability of a bee flying different distances is different, and a larger probability near the center corresponds to a bee. Then we can represent the probability of a bee appearing at any point (x, y) by the one-dimensional, two-dimensional normal distribution model and the linear decreasing model.

6.2.1 One-dimensional normal distribution of bees

First, we know that the probability of a bee appearing at any location is related to the distance d. Therefore, we define the bee probability density function based on a normal distribution:

$$p(d)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{d^2}{2\sigma^2}}$$

Finally, as above, this probability is divided by the circumference of its corresponding circle, i.e. the probability of appearing at the point (x,y) is:

$$p(x,y)=\frac{2p(d)}{2\pi d}=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{d^2}{2\sigma^2}}\times\frac{1}{\pi d}$$

The diagram is shown below:Let $\sigma=\frac{1}{2}$, then the possibility function (7.6) is:

$$\mathrm{p(x,y)}=\frac{2\mathrm{p(d)}}{2\pi \mathrm{d}}=\frac{2}{\sqrt{2\pi}}\mathrm{e}^{-2(\mathrm{d})^2}\times\frac{1}{\pi \mathrm{d}} \quad (7.6)$$

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Hence, we can calculate the probability that the bees are at any location. In order to fit the actual situation, we examine that farmland is generally rectangular, so we can assume that the 81,000 m2 farm is square, and the green area in the Fig. 2 below is the farm:

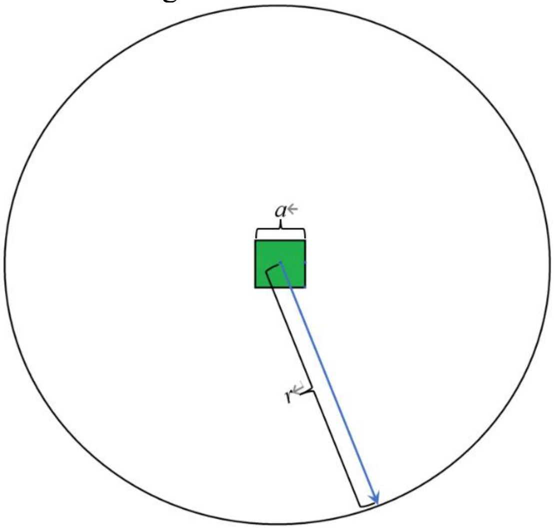

Fig.2 Farm Legend

Since the specific location of the beehives in the farm is not specified in the assumption, we can assume several locations for discussion and finally choose the more reasonable value:

Assuming that the hive is in the middle of the farm, according to the area of the farm is 81000 square meters, the vertex is at a distance from the center, which is:

$$d_1 = 0.5\sqrt{81000} \approx 201m$$

By substituting this value into equation (7.6), we can get $P_1 = 0.00038$. Since the population of bee colony N * Possibility at the outer shell P1 > number of bees per unit area α > 3948, $N >$ 3948. Hance, $H > N/3000 \approx 1.31$ according to equation (7.5). Thus, at least two Beehives is required in this situation.

Assuming that the hive is at one of the vertices of the farm, since we need to ensure that the canola flowers farthest 1m2 from the farm can also be pollinated by the bees, substituting the farthest distance d2=$\sqrt{81000} \approx 402$ into equation (7.6) yields P1.

$$P_1 \approx 8 \times 10^{-16}$$

Since the population of bee colony N * Possibility at the outer shell P1 > number of bees per unit area α, N > 1.875×10^{14}. Therefore, according to the equation (7.5), $H > N/3000 \approx 6.25 \times 10^{10}$. In this case, 6.25×10^{10} Beehives are required.

Assuming d = 300, substitution into equation (5.6) gives P1≈ 1.28×10^{-9}. Since the population of bee colony N * Possibility at the outer shell P1 > number of bees per unit area α, N $> 1 \times 10^{8}$. According to the equation (7.5), H> $N/3000 \approx 33334$. Therefore, 33334 Beehives are required.

6.2.2 Two-dimensional normal distribution of bees

The two-dimensional normal distribution probability density function is as follows:

$$f(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\{\frac{1}{2(1-\rho^2)}[\frac{(x-\mu_1)^2}{\sigma_1^2} - 2\rho\frac{(x-\mu_1)(x-\mu_2)}{\sigma_1\sigma_2} + \frac{(x-\mu_2)^2}{\sigma_2^2}]\}}$$

Since the probability of a bee flying in each direction is equal, we can let σ1=σ2=2. The shape of the image of this function is approximately as follows: When the farthest distance from the farm, which is at the vertex of the farm, has coordinates (142, 142), and the probability at this point is: P2≈ 4×10^{-6}. Since $N \times P_2 > \alpha, N \approx 37500$. Hance, according to equation (7.5),

$$H > N/3000 = 12.5$$

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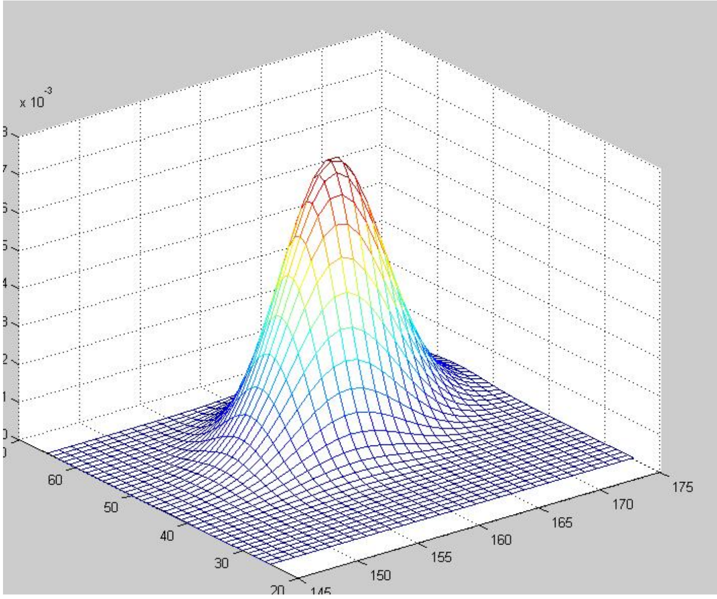

Fig.3 The two-dimensional normal distribution probability density function is as follows

In that case, about 13 Beehives are needed.

7. Strength and Weakness

7.1 Strength of Our Model

The strength of our established retrofit program evaluation model are mainly: A scientifically valid method for measuring covered area based on binarized images was developed. Multiple models were developed to study bee density and land area, and were discussed by selecting σ from multiple one- and two-dimensional normal distribution models, assuming multiple hive locations on the farm to ensure wide coverage and realistic conditions.

7.2 Weakness of Our Model

First of all, the weakness of our model is mainly in terms of data accuracy. The data used may contain errors. Additionally, another weakness of our research work is that the model does not take into account the optimal solution. Due to the time constraint, we were not able to design a more efficient method to calculate the scoring results for various combinations of scenarios to find the optimal solution. Finally, for the one- and two-dimensional normal distribution models constructed above, we were unable to find a specific basis for the determination of σ in the models and failed to build a more accurate model scientifically and rationally.

8. Conclusion

In order to better study the needs of honey bee populations for farm pollination, our task was to determine the number of honey bee colonies over time and find the factors that have the greatest impact on honey bee populations and the number of hives needed on farms. We set the crop in the farm as canola and assumed the farm to be square to fit the real situation. To study the number of hives in the farm, we discussed both the number of bees needed per unit area of farmland and the distribution of bees. We first obtained the density of bees by establishing a functional relationship between canola density, flowering period, etc., and then by constructing a normal distribution model including one-dimensional, two-dimensional and linear decreasing equations, and discussed about the location of the hives in the farm and discarded the unreasonable results. The optimal density of bees in the outermost layer of the farm with guaranteed pollination was determined, and the final number of hives required for this farm was derived.

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