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An improved model to simulate the effective material properties of functionally graded porous plates

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Abstract. This paper develops an improved model to estimate the effective material properties of functionally graded porous plates with evenly and unevenly distributed pores. Based on the higher-order shear deformation plate theory and extended von Kármán-type equations, dynamic equilibrium equations of the plate were built with the effect of the viscoelastic foundation. By using a two-step perturbation technique, the nonlinear frequency and dynamic response were obtain. The effects of the pore volume fraction, porosity distributions, and ceramic mass fraction were investigated.

Keywords. Perturbation technique; Nonlinear vibration; Functionally graded plate; Pores; Viscoelastic foundation

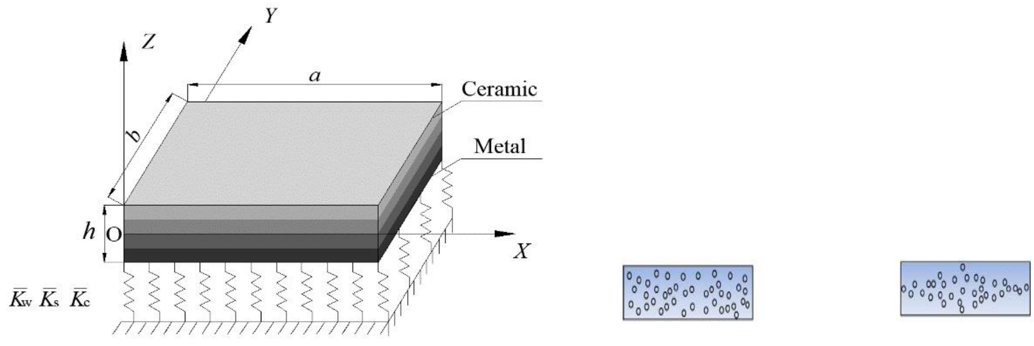
1. Introduction

Due to the imperfect technique of fabricating functionally graded materials (FGMs), internal pores have been detected in the materials[1]. Furthermore, the pores in the middle are more than in the other areas[2]. This was because a secondary material was difficultly infiltrated into the middle area. In contrast, it was easily infiltrated into the upper and lower areas. To study the effect of internal pores on dynamics characteristics, the free and forced vibrations of FGM plates with internal pores were investigated by Rezaei[3] and Wang[4, 5]. They found that the pores have significant influences on the natural frequencies and transient responses. In fact, their results are not accurate enough, because the pore volume fraction was supposed to be small and omitted in calculating the total volume of the plates.

To eliminate the assumption, an improved model was presented to reckon the effective properties of the materials in the present research. Two kinds of porosity distributions, even and uneven, were considered. In the framework of Reddy's plate theory[6] and extended von Kármán-type equations[7], the dynamic equilibrium equations of a porous FGM on viscoelastic foundation were built and solved by using the two-step perturbation technique[8]. The influences of the internal pores on the nonlinear vibration and transient deflection responses were discussed.

2. Theoretic Formulations

Figure 1 shows a rectangular porous FGM plate. The plate is made of metal and ceramics and accompanied by evenly distributed (ED) or unevenly distributed (UD) pores. The composite materials change continuously from the bottom to the top in the thickness direction(z).



(a) (b)

Figure 1. (a) FGM plate on viscoelastic medium (b) ED and UD pores

In the previous research, the pore volume fraction α was assumed to be small ($\alpha << 1$). Hence, it was omitted in the total volume of the plate,i.e. $V_{\mathrm{c}} + V_{\mathrm{m}} = 1$, in which V_{c} and V_{m} denote the volume fractions of the ceramic and fraction. Evidently, the assumption leads to inaccurate results.

To eliminate the assumption, we let $V_{\mathrm{c}} + V_{\mathrm{m}} + \alpha = 1$ and $W_{\mathrm{c}} + W_{\mathrm{m}} = 1$, in which $\mathrm{W_c}$ and $\mathrm{W_m}$ are the mass fractions of the ceramic and metal. Thus, V_c can be calculated as follow:

$$V_{\mathrm{c}} = (1 - \alpha)\frac{W_{\mathrm{c}}}{W_{\mathrm{c}} / \rho_{\mathrm{c}} + W / \rho_{\mathrm{m}}}, \tag{1}$$

where ρ_{c} and ρ_{m} are the mass densities of the ceramic and metal.

Suppose the ceramic materials follow the power function distribution in the thickness direction. The ceramic volume distribution V_{c}^* can be stated as:

$$V_{\mathrm{c}}^*(Z) = V_{\mathrm{c}}\left(\frac{Z + 2h}{2h}\right)^N, \tag{2}$$

where N is the material volume index.

By using an improved rule of mixture, the effective material property P, such as Young's modulus, mass density and Poisson ratio can be stated as:

$$P = P_{\mathrm{c}} V_{\mathrm{c}}^* + P_{\mathrm{m}}\left(1 - V_{\mathrm{c}}^* - \alpha^*\right), \tag{3}$$

where the porosity distribution α^* is assumed to be :

$$\alpha^* = \alpha \quad \text{(ED pores)}, \tag{4}$$

$$\alpha^* = \alpha\left(1 - \frac{2|Z|}{h}\right) \quad \text{(UD pores)}. \tag{5}$$

In the framework of Reddy's plate theory[6] and the extended von Kármán-type equations[7, 8], the dimensionless dynamic equilibrium equations of the plate can be stated by using the displacements ($\widetilde{W}$, $\widetilde{\varphi}_x$, $\widetilde{\varphi}_y$)and stress function $\widetilde{F}$:

$$\begin{aligned} &L_1(\widetilde{W}) - L_2(\widetilde{\varphi}_x) - L_3(\widetilde{\varphi}_y) + \gamma_1 L_4(\widetilde{F}) + K_w \widetilde{W} - K_s \nabla^2 \widetilde{W} + K_c \frac{\partial \widetilde{W}}{\partial t} \\ &= \gamma_1 \beta^2 L(\widetilde{W}, \widetilde{F}) + L_5(\ddot{\widetilde{W}}) + \gamma_2 \frac{\partial \ddot{\widetilde{\varphi}}_x}{\partial x} + \gamma_2 \beta \frac{\partial \ddot{\widetilde{\varphi}}_y}{\partial y} + \widetilde{\lambda}_q , \end{aligned} \tag{6}$$

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$$L_6(\widetilde{F})+\gamma_3 L_7(\widetilde{\varphi}_x)+\gamma_3 L_8(\widetilde{\varphi}_y)-\gamma_3 L_9(\widetilde{W})=-\frac{1}{2}\gamma_3\beta^2 L(\widetilde{W},\widetilde{W}), \tag{7}$$

$$L_{10}(\widetilde{W})+L_{11}(\widetilde{\varphi}_x)-L_{12}(\widetilde{\varphi}_y)+\gamma_1 L_{13}(\widetilde{F})=\gamma_4\frac{\partial\ddot{\widetilde{W}}}{\partial x}+\gamma_5\ddot{\widetilde{\varphi}}_x, \tag{8}$$

$$L_{14}(\widetilde{W})-L_{15}(\widetilde{\varphi}_x)+L_{16}(\widetilde{\varphi}_y)+\gamma_1 L_{17}(\widetilde{F})=\gamma_4\beta\frac{\partial\ddot{\widetilde{W}}}{\partial y}+\gamma_5\ddot{\widetilde{\varphi}}_y, \tag{9}$$

in which the dots mean the derivative corresponding to time. The linear operators $L_i()$, nonlinear operators $L()$, constants γ_i and β, and dynamic load $\lambda_q(x,y,t)$ were given in previous reported[9]. K_w is the dimensionless Winkler foundation parameter, K_s is the Pasternak foundation parameter, and K_c is the viscous foundation parameter.

The four edges of the plate are assumed to be simply supported. The boundary conditions are given as

$$x=0,\ \pi:\ \widetilde{W}=\widetilde{\varphi}_y=\frac{\partial^2\widetilde{F}}{\partial x\partial y}=0, \tag{10}$$

$$y=0,\ \pi:\ \widetilde{W}=\widetilde{\Psi}_x=\frac{\partial^2\widetilde{F}}{\partial x\partial y}=0. \tag{11}$$

3. Solution Procedure

For the sake of solving the governing equations (6)-(11), a two-step perturbation technique[7, 8] is used in present research. The asymptotic solutions are assumed as

$$\begin{aligned}
&\widetilde{W}(x,y,\bar{\tau},\varepsilon)=\sum_{i=1}\varepsilon^i\widetilde{w}_i(x,y,\bar{\tau}),\ \widetilde{F}(x,y,\bar{\tau},\varepsilon)=\sum_{i=1}\varepsilon^i\widetilde{F}_i(x,y,\bar{\tau}),\\
&\widetilde{\varphi}_x(x,y,\bar{\tau},\varepsilon)=\sum_{i=1}\varepsilon^i\widetilde{\varphi}_{xi}(x,y,\bar{\tau}),\ \widetilde{\varphi}_y(x,y,\bar{\tau},\varepsilon)=\sum_{i=1}\varepsilon^i\widetilde{\varphi}_{yi}(x,y,\bar{\tau}),\\
&\widetilde{\lambda}_q(x,y,\bar{\tau},\varepsilon)=\sum_{i=0}\varepsilon^i\widetilde{\lambda}_i(x,y,\bar{\tau}),
\end{aligned} \tag{12}$$

In equation (12), the time parameter $\bar{\tau}$ ($\bar{\tau}=\varepsilon\tau$) is adopted to improve the perturbation procedure. Substituting the equation (12) into the equations (6)-(11), then solving the perturbation equations length by length, the displacements $\widetilde{W}$, $\widetilde{\varphi}_x$ and $\widetilde{\varphi}_y$ can be obtained. The transverse load is also derived as

$$\begin{aligned}
\widetilde{\lambda}_q(x,y,\tau)=&[g_1\varepsilon\widetilde{w}_1(\tau)+K_c\varepsilon\dot{\widetilde{w}}_1(\tau)+g_2\varepsilon\ddot{\widetilde{w}}_1(\tau)]\sin mx\sin ny+(\varepsilon\widetilde{w}_1(\tau))^2(g_3\cos 2mx\\
&+g_4\cos 2ny)+\bar{\alpha}g_5(\varepsilon\widetilde{w}_1(\tau))^3\sin mx\sin ny)+O(\varepsilon^3).
\end{aligned} \tag{13}$$

where g_i (i=1-5)are load coefficients. If the free vibration is considered, the constant $\bar{\alpha}$ is 1, otherwise $\bar{\alpha}$ is zero. Multiplying equation (13) by ($\sin mx \sin ny$) and integrating over the plate area, yield the nonlinear ordinary differential equation:

$$g_2\frac{d^2(\varepsilon\widetilde{w}_1)}{d\tau^2}+K_\mathrm{c}\frac{d(\varepsilon\widetilde{w}_1)}{d\tau}+g_1(\varepsilon\widetilde{w}_1)+g_3'(\varepsilon\widetilde{w}_1)^2+\bar{\alpha}g_5(\varepsilon\widetilde{w}_1)^3=\widetilde{\lambda}_q(\tau), \tag{14}$$

in which

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$$\tilde{\lambda}_q(\tau)=\frac{4}{\pi^2}\int_0^{\pi}\int_0^{\pi}\lambda_q(x,y,\tau)\sin mx\sin ny\,dxdy\,. \tag{15}$$

By solving the equation (14), the nonlinear frequency and transient responses can be calculated.

4. Results and Discussion

The several non-dimensions parameters are used as follows:

$$k_{\mathrm{w}}=\frac{\overline{K}_w a^4}{D_{\mathrm{m}}},k_{\mathrm{s}}=\frac{\overline{K}_s a^2}{D_{\mathrm{m}}},k_{\mathrm{c}}=\frac{\overline{K}_c a^3}{\pi^3 D_{\mathrm{m}}}\sqrt{\frac{E_c}{\rho_{\mathrm{c}}(1-\nu_c^2)}},D_{\mathrm{m}}=\frac{E_{\mathrm{m}}h^3}{12(1-\nu_{\mathrm{m}}^2)},$$

$$D_{\mathrm{c}}=\frac{E_{\mathrm{c}}h^3}{12(1-\nu_{\mathrm{c}}^2)},\Omega=100\overline{\omega}_L h\sqrt{\frac{\rho_{\mathrm{m}}}{E_{\mathrm{m}}}},\hat{\Omega}=\frac{\overline{\omega}_L a^2}{h}\sqrt{\frac{\rho_{\mathrm{m}}(1-\nu_{\mathrm{m}}^2)}{E_{\mathrm{m}}}}\,. \tag{16}$$

4.1 Comparison Study

The fundamental frequencies $\hat{\Omega}$ of an FGM plate, accompanied by ED and UD pores, are listed in Table 1 . As shown in this Table, the discrepancies between reported[3] and the present results were small.

Table 1 Fundamental frequencies Ω for a porous FGM plate.

α	Method	N=0		N=0.1		N=0.5		N=1.0	
		ED	UD	ED	UD	ED	UD	ED	UD
0.2	Ref.[3]	3.003	2.999	2.875	2.884	2.456	2.524	2.104	2.473
	Present	2.948	2.981	2.823	2.868	2.412	2.509	2.076	2.233
	Error(%)	1.87	0.60	1.84	0.55	1.79	0.59	1.33	0.97

4.2 Parametric Studies

The influences of various material and pore parameters are discussed. The thickness, length, and width of the porous FGM plate are 0.1m, 1.0 m, and 1.0 m, respectively. The material parameters are E_{c}=3.223 × 10^{11} Pa, ρ_{c}=2.370 g/cm^3 and ν_{c}=0.24 for Si_3N_4, and E_{m}=2.078 × 10^{11} Pa, ρ_{m}= 8.166 g/cm^3, and ν_{m}=0.3178 for SUS304.

Tables 2-3 list the dimensionless fundamental frequency $\hat{\Omega}$ with different values of ceramic mass fraction W_{c} , pore volume fractions α, and material index N. The two tables show that the dimensional frequency rises as the ceramic mass fraction W_{c} increases, whereas it is decreased with the rising parameter N. The tables also show that the dimensional frequencies for ED pores are different from those for UD pores. Moreover, The dimensional frequency is not always decreased with the rise of pore parameter α, because the pores can weaken the effective stiffness and mass of the plate simultaneously. If the effect of pores on the stiffness is more significant than that of mass, the dimensionless frequency is decreased. Otherwise, it is increased.

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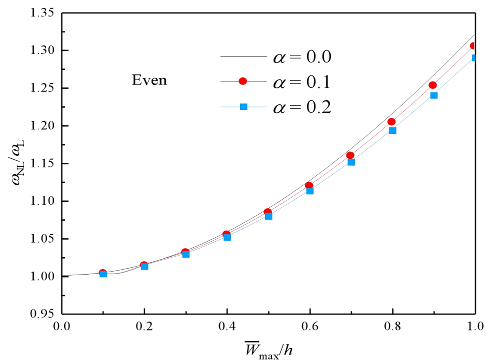
Table 2 Fundamental frequency $\hat{\Omega}$ for even porosity distribution.

k_w	k_s	k_c	w_c	α	N			
					0.0	0.1	0.5	1.0
100	10	10	0.1	0.0	8.1930	8.1295	7.9826	7.9081
				0.1	8.3021	8.2409	8.0994	8.0281
				0.2	8.4494	8.3905	8.2547	8.1866
			0.3	0.0	9.2080	9.0421	8.6219	8.3888
				0.1	9.2818	9.1208	8.7142	8.4897
				0.2	9.3915	9.2355	8.8434	8.6280

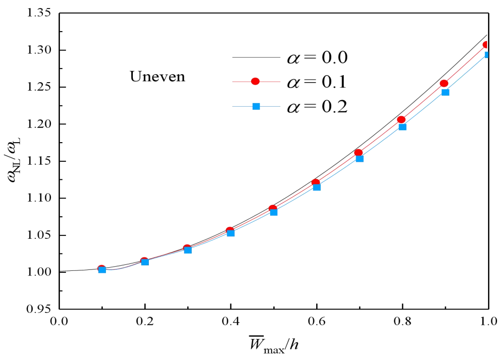
Table 3 Fundamental frequency $\hat{\Omega}$ for uneven porosity distribution.

k_w	k_s	k_c	w_c	α	N			
					0.0	0.1	0.5	1.0
100	10	10	0.1	0.0	8.1930	8.1295	7.9826	7.9081
				0.1	8.2740	8.2226	8.0788	8.0098
				0.2	8.3644	8.3100	8.1850	8.1220
			0.3	0.0	9.2080	9.0421	8.6219	8.3888
				0.1	9.2262	9.0700	8.6764	8.4595
				0.2	9.2487	9.1109	8.7378	8.5388

Fig.2 reveals the effect of the pore parameter α on the ratio of nonlinear to linear frequency. It can be observed that the frequency ratio rises as the parameter α increases.



(a) ED pores



(b) UD pores

Figure 2 Effect of pore volume fraction α on frequency ratio.

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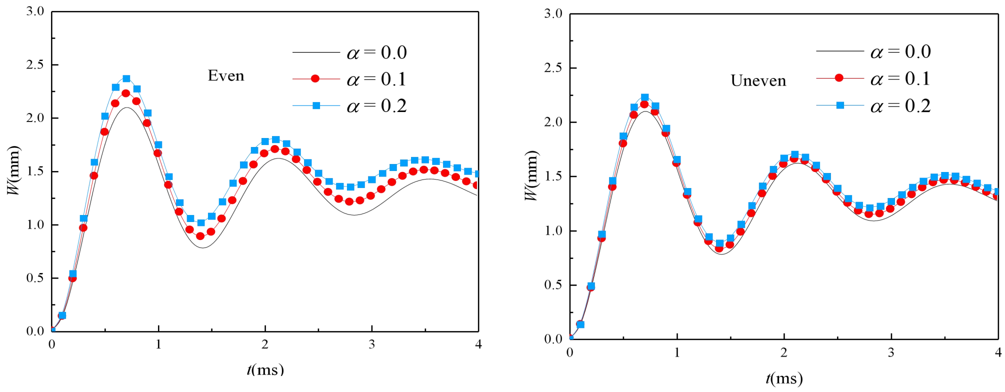
The influence of pore parameters α on transient deflections was shown in Fig.3. As expected, the amplitude is raised as both the pore parameter α and material index N increase. The effect of ED pores is more significant than that of UD pores.



(a) ED pores (b) UD pores

Figure 3 Effect of pore volume fraction α on transient deflection.

5. Concluding Remarks

In this paper, an improved model to reckon the material properties of porous FGM plates was given. The procedure of analyzing the nonlinear vibration were presented and used to study the effects of pores and visoelastic foundations. The numerical results showed that the natural frequency and the ratio of nonlinear to linear frequency were decreased by increasing pore volume fraction. Conversely, the amplitude of dynamic deflection was increased.

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