

Numerical Simulation of Influence of Prestress on Rock Crack Propagation

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Abstract. Based on the damage evolution law of energy, through the three-point bending test of granite, the calculation formula of three-point bending fracture energy of rock dominated by type I fracture failure is calculated, and the influence of dimensionless prefabricated crack length and prestress on rock fracture failure caused by crack propagation is studied. The results show that with the increase of dimensionless prefabricated crack length, the peak load of granite increases, but its fracture toughness increases first and then decreases. The size and application of prestress have different effects on crack propagation in rock samples. It is proposed that the influence factor of prestress is used to characterize the strength of crack propagation inhibition. In engineering construction, high strength prestressed anchor bolts and expanded support area can be adopted to prevent collapse accidents caused by excessive deformation of surrounding rock.

Keywords: extended finite element; fracture energy; loading method; crack tip ; prestress influencing factor.

1. Introduction

The deep rock is in the initial state of high ground stress, which causes the internal crack defects of brittle rock in an initial damage state. When subjected to different external dynamic disturbances, different dynamic responses occur in the rock. For the same rock subjected to different initial stress, the initial crack damage state in the rock will be different. When the same dynamic disturbance acts on the rock, the dynamic response and fracture failure of the rock are also different. The study of dynamic mechanical properties of prestressed rock has important value for the stability evaluation of rock excavation and surrounding rock support in deep underground engineering.

Hong-lin Mi [1] analyzed that different rock types have different crack evolution paths; Kui Zhao [2] studied the influence of rock particle size composition on rock tensile failure; Hui-zi Zhao [3] studied the influence of different incision positions on rock fracture failure; Ying-jie Li [4] and Nasseri [5] studied the effect of bedding on rock fracture mechanism and fracture criterion; Jian-ping Zuo [6] studied the change of mechanical parameters and fracture failure behavior of rock after heat treatment. Xiao-ran Wang [7] studied the nonlinear mechanical behavior of crack tip propagation in coal samples.

The above studies studied the fracture failure characteristics of rock from the aspects of rock type, composition and defect location. However, the deep rock is in a complex tectonic stress, and the crack propagation and fracture failure of rock are related to its stress state.

Wu-xing Wu [8] studied that high static prestress is the premise and main factor of rock weakening; Feng-qiang Gong [9] studied the effect of different pre-static load on rock fracture toughness; Kun Du [10] and Kai-wen Xia [11] studied the influence of prestressed path on rock failure mode; Zhi-hua Zhou [12] studied the mechanical response of prestressed rock under the coupling effect of geostress and seepage water pressure; Tao Zhou [13] and Weng [14] showed that the axial prestress has great influence on the fracture behavior of rock. Jia-wei Gong [15] reveals the influence mechanism of prestress on rock freeze-thaw damage.

The influence of pre-stressed path and loading direction on rock fracture failure is summarized. Few scholars have studied the influence of prestress on rock internal crack propagation from the microscopic point of view, which leads to rock fracture failure. In-depth study of the influence mechanism of prestress on rock crack propagation has important engineering significance for deep surrounding rock excavation and advanced support treatment.

2. Foundations of Theoretical Research

2.1 Extended Finite Element Theory

Professor Belyschko of the United States first proposed to use the extended finite element to deal with discontinuous problems, and the unit decomposition method is the core principle of the extended finite element. Based on the displacement mode of the traditional finite element, the discontinuous displacement of the elements on both sides of the crack penetration and the enrichment function of the element at the crack tip are added to construct the displacement field. Belyschko decomposes the basic element based on the interpolation function, and the expression of the crack surface displacement interpolation function is as follows [16]:

$$u(x) = \sum_{k \in I} N_k(x) u_k + \sum_{j \in I} \bar{N}_k(x) H(x) a_k \quad (1)$$

In the formula, N is the finite element node set, I is the crack complete cutting element node set, $H(x)$ is the jump function, $H(x)=1$ on the crack surface, $H(x)=-1$ on the lower side of the crack surface, is the displacement function related to $H(x)$ and k point, is the additional degree of freedom of k point.

It is the additional displacement field function of the crack tip, which reflects the singularity of the crack tip.

$$\phi_\alpha(x) = \sqrt{r} \left[\sin \frac{\theta}{2}, \cos \frac{\theta}{2}, \sin \frac{\theta}{2} \cos \theta, \sin \frac{\theta}{2} \cos \theta \right] \quad (2)$$

where r and θ represent the polar coordinates of crack tip.

Belyschko put forward the level set function as displacement function (shape function), its basic form is:

$$\varphi(x, t) = \pm \min_{x_i \in \Gamma(t)} \|x - x_i\| \quad (3)$$

$$g_i = (x - x_i) g \frac{U_i}{\|U_i\|} \quad (4)$$

$\varphi(x, t)$ and g_i respectively describe the positions of cracks and crack tips in the unit.

2.2 Implementation of Extended Finite Element in Software

The theory of extended finite element is introduced in detail. Taking ABAQUS software as an example, the method of simulating rock crack propagation by extended finite element is introduced.

In the process of ABAQUS calculation, the description of discontinuous field is independent of the grid boundary, and the shape function basis is constructed by using the known analytical solution. The general Gauss integral of the element will cause the solution to not meet the error range. Different high order integral methods can obtain more accurate solutions regardless of the thickness of the grid.

Rock three-point bending is dominated by type I fracture failure. According to the calculation formula of rock three-point bending stress intensity factor, the calculation formula of energy consumption G_I for unit crack propagation of rock type I fracture is calculated. The following formula (6) is used to judge the fracture damage of materials by VUMAT subroutine.

$$\alpha = \frac{a}{W} \quad (5)$$

$$G_I = \frac{9 \alpha P_{\max}^2 S^2}{4 E (1 + 2 \alpha) B^2 (1 - \alpha)^3 W^3} * \left[1.99 - \alpha (1 - \alpha) (2.15 - 3.93 \alpha + 2.7 \alpha^2) \right]^2 \quad (6)$$

In the formula: α is dimensionless prefabricated crack length, W is sample height / mm, a is prefabricated crack length / mm, E is rock elastic modulus / GPa, B is sample width / mm, S is span / mm, P_{\max} is the rock type I fracture peak load kN.

3. Numerical Simulation of Three - point Bending of Rock

The rectangular model of granite with length \times width \times height of 100 mm \times 25 mm \times 50 mm is established in the Part module of ABAQUS software. The net span S is 80 mm, a crack is prefabricated in the middle of the span, and the initial crack height is h mm. There are two loading modes of prestress (upper section loading and full section loading), as shown in Figure 1.

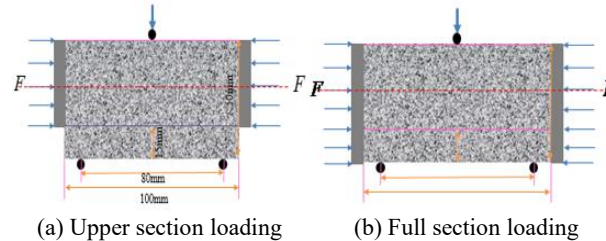


Figure 1. Lateral stress loading mode

Simple support constraint is set at the lower end of granite member, and the distance between the two constraints is 80 mm. Displacement constraint in the vertical direction is adopted, and displacement loading control is adopted. The loading rate is 0.01 mm / min. Granite rock material parameters such as table 1.

Tabla 1 basic physical and mechanical properties of granite

Elastic modulus /GPa	Poisson ratio	Uniaxial tensile stress /MPa	Maximum principal stress /MPa	Density / kg/m3
73.82	0.24	10	10	2620

The influence of dimensionless prefabricated crack length on three-point bending fracture failure of granite was explored. The dimensionless prefabricated crack length was set as 0.1, 0.2, 0.3, 0.4 and 0.5, respectively. The stress intensity factors were 1.49, 1.45, 1.54, 1.77 and 2.32 MPa $\cdot \sqrt{m}$.

The influence of prestress size and prestress loading mode on three-point bending failure of granite was studied. The dimensionless crack length was controlled and the prestress size was set to 4 gradients, which were 1 MPa, 3 MPa, 5 MPa and 7 MPa, respectively. The loading mode was full section loading and upper end face loading.

4. Analysis of Simulation Results

4.1 Influence law of dimensionless prefabricated crack length

By simulating the three-point bending fracture failure process of granite rock samples, as shown in Fig. 2, the peak loads of granite rock samples under five groups of tests are 1.97 kN, 1.50 kN, 1.19 kN, 0.98 kN and 0.79 kN, respectively. Using the calculation formula of rock fracture toughness(KIC) [17], the fracture toughness values of rock samples are calculated as follows: 2.15, 2.01, 1.82, 1.67 and 1.51 MPa $\cdot \sqrt{m}$.

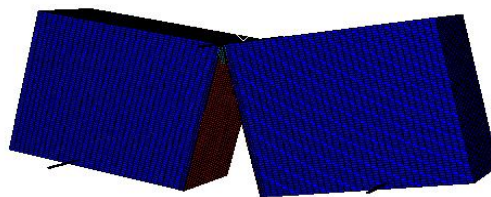


Figure 2. Crack propagation diagram of rock sample

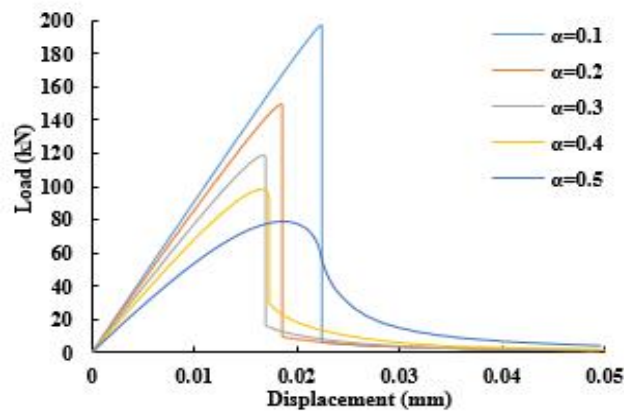


Figure 3. Load displacement relation curve

It can be seen from Fig. 3 that with the increase of crack length, the peak value of the curve gradually decreases, and the peak value is the instability load of the rock sample. Under short-term loading, the rock sample is in the elastic deformation stage, and the crack has not begun to expand upward. The stress field stress at the crack tip is too large under continuous loading. The tensile stress of the rock sample unit reaches the cracking failure strength, and the crack is expanded. The granite rock sample is a brittle material, so the load decreases rapidly.

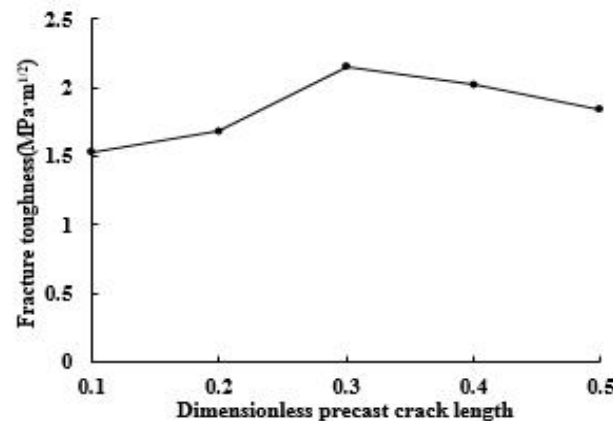


Figure 4. Dimensionless precast crack length fracture toughness relationship

It can be seen from Fig. 4 that the fracture toughness of rock increases first and then decreases with the increase of crack length. There is a turning point in the relationship curve between dimensionless prefabricated crack length and fracture toughness. Under the calculation conditions, the maximum value of rock fracture toughness is obtained at the turning point. When h is small, the micro-cracks in rock samples will develop after loading. When the stress intensity factor at the crack tip reaches the rock failure strength, the crack will propagate upward. When the prefabricated artificial fracture length is longer, the energy and load required for the fracture failure of rock samples are smaller, and the measured fracture toughness is smaller.

4.2 Influence Mechanism of Prestress Size

The numerical simulation test results of granite under four prestress gradients are analyzed, and the fracture toughness of rock samples under different prestress conditions is calculated, such as figure 6.

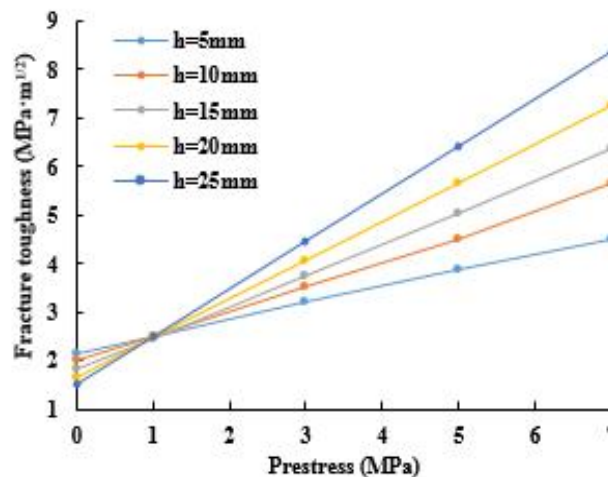


Figure 5. Relationship between lateral stress and fracture toughness

It can be seen from Fig. 5 that the fracture toughness of rock samples increases with the increase of prestress value. In the initial loading stage, the prestress leads to the shrinkage and closure of cracks in the rock sample, which leads to the increase of rock stiffness. The partial tensile stress in the unit will be balanced after the pre-stress of the rock sample is applied to avoid the phenomenon of tensile stress concentration. The pre-stress affects the upward movement of the crack tip and hinders the crack propagation, which fully indicates that the pre-stress in the three-point bending test of granite can inhibit the crack propagation. Setting pre-stressed anchor in deep rock excavation, pre-stressed anchor and surrounding rock constitute anchor solid, which is beneficial to avoid confining pressure collapse and other accidents.

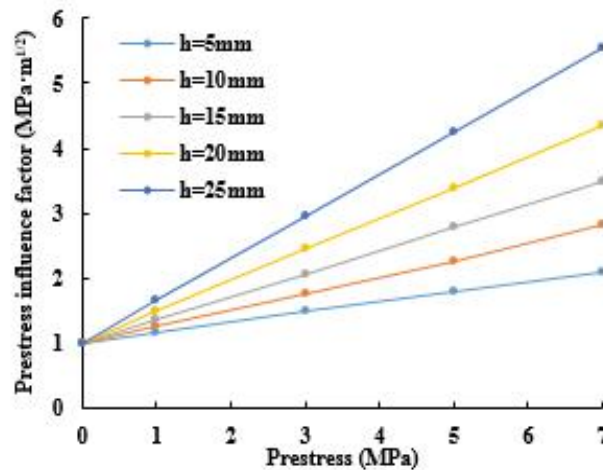
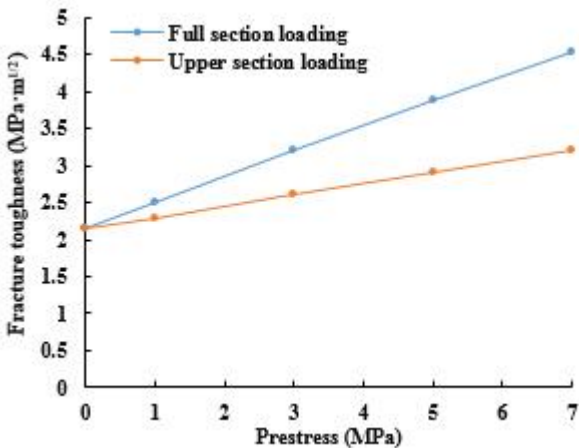


Figure 6. Relationship between lateral stress and lateral stress influencing factors

The influence factor of prestress is the ratio of fracture toughness of rock sample with different prestress and without prestress. The size of prestress influence factor can reflect the ability of rock samples to resist crack propagation. It can be seen from Fig. 6 that with the increase of prestress, the influence factor of prestress increases, indicating that the inhibitory effect of prestress on crack propagation is significantly enhanced.

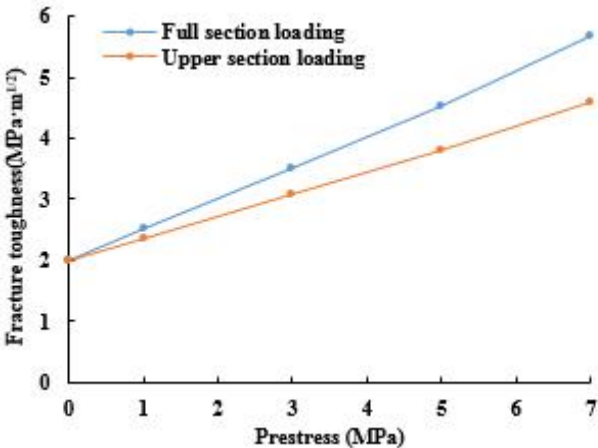
4.3 Action Law of Prestressed Loading Mode

There are two ways to apply prestress, namely full section loading and upper section loading, as shown in Fig. 1. In the previous study, pre-stressing on the whole section can inhibit crack propagation. The influence of different loading methods on the crack propagation of granite rock samples is studied by numerical simulation technology, and Fig. 7.



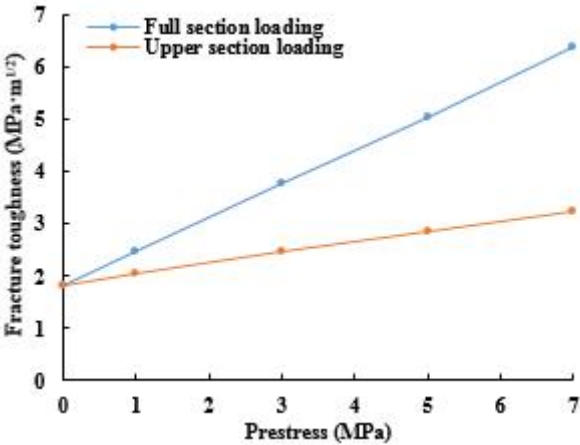
(a) $\alpha=0.1$

Full section loading: $y=0.339x+2.100$
Upper section loading: $y=0.149x+2.149$



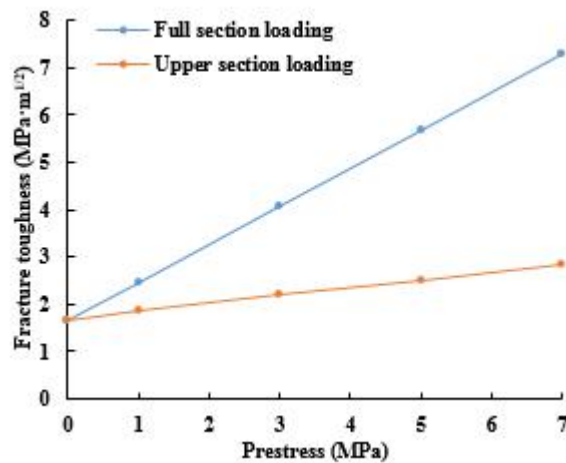
(b) $\alpha=0.2$

Full section loading: $y=0.518x+1.988$
Upper section loading: $y=0.366x+2.001$



(c) $\alpha=0.3$

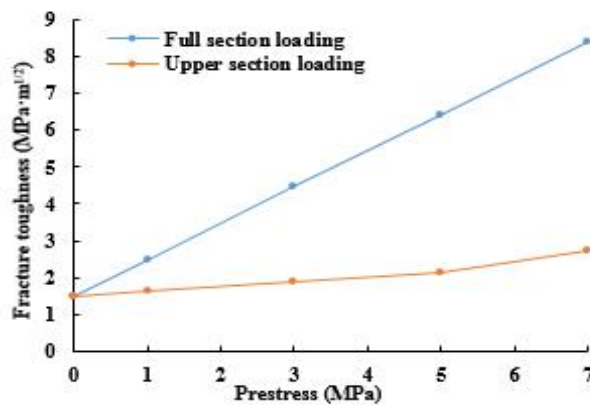
Full section loading: $y=0.648x+1.815$
Upper section loading: $y=0.201x+1.829$



(d) $\alpha = 0.4$

Full section loading: $y = 0.799x + 1.669$

Upper section loading: $y = 0.166x + 1.676$



(e) $\alpha = 0.5$

Full section loading: $y = 0.982x + 1.509$

Upper section loading: $y = 0.165x + 1.461$

Figure 7. Relationship between loading mode and fracture toughness

Figure 7. shows that the effect of full section loading on crack propagation is greater than that of upper section loading in three-point bending test of granite rock samples. In the test, the location of the crack is on the tensile side. Prestressed full section loading reduces the tensile stress of the attachment element at the crack tip, which can effectively hinder the movement of the crack tip. When $\alpha = 0.5$ and prestress is 7 MPa, the fracture toughness of rock sample under full section prestress is 4 times that of rock sample under upper section prestress, which fully indicates that rock mass excavation can improve the stability of surrounding rock by expanding the support surface area in practical engineering..

5. Conclusion

Three-point bending numerical simulation test of granite is carried out to reveal the influence of dimensionless prefabricated crack length and prestress on crack propagation and fracture failure of rock samples. The following conclusions are obtained :

(1) The KIC of rock samples increases first and then decreases with the increase of dimensionless prefabricated crack length. Under the calculation condition, the KIC reaches the maximum when the crack length is 0.3.

(2) Prestress restrains crack propagation of rock sample. With the increase of prestress, the fracture toughness of rock sample increases, and the influence factor of prestress increases, showing a stronger inhibitory effect on crack propagation. In practical engineering, prestressed anchor with high strength can be adopted to prevent the collapse of surrounding rock deformation.

(3) Both the pre-stress loading on the upper section and the full section will inhibit the crack propagation, but the effects of different pre-stressing methods on the crack propagation are different. As the pre-stress increases to a certain value, the influence factor of the full section pre-stress is about four times that of the upper section pre-stress, and the effects of the two methods on inhibiting the crack propagation are very different. It is suggested that the stability of surrounding rock should be improved by expanding the supporting area in the engineering.

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