Dynamic Application of Matlab in Teaching of Hyperbolic Paraboloid

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Abstract. In teaching of hyperbolic paraboloid, this paper breaks through the traditional teaching method. Through its theoretical analysis, combined with graphic visualization of MATLAB software, this paper shows two formation processes and notch transformation processes of hyperbolic paraboloid, so as to enhance students' learning interest, cultivate students' spatial imagination and improve teachers' teaching effect, so as to further promote the modernization of basic course teaching methods.

Keywords: hyperbolic Paraboloid; Matlab soft; Dynamic Application

1.Introduction

Analytical Spatial Geometry is one of the three basic courses of mathematics majors in colleges, and it is the extension and promotion of mathematics in high schools, it is a bridge between elementary mathematics and advanced mathematics[1-2]. However, teacher's teaching method is still relatively traditional, and formation process of curved surface is still displayed through teachers' lectures or static diagrams, so that students cannot dynamically understand the formation process of geometric figures. In addition, lack of imagination and information extraction ability make students less interested in this course, let alone imagine the formation process of hyperbolic paraboloid figure[3].

With the rapid development of science and technology, modern educational technology is more used in mathematics teaching. The emergence of these tools not only provides an intuitive display for teaching of abstract content, but also helps students to further cultivate their spatial imagination. In mathematics classrooms, tools such as Flash, Geometric Sketchpad, and Matlab software are often used to assist teaching. Matlab is a relatively simple language developed by Mathworks that integrates three basic functions of numerical computation, symbolic computation and graphic visualization, it has the functions of visualization of two-dimensional and three-dimensional data, graphics processing and graphic animation [4].

Therefore, in teaching of hyperbolic paraboloid, this paper uses visualization and animation functions of graphics with help of Matlab software to dynamically display the formation process and change process of hyperbolic paraboloid, so as to improve students' learning interest and cultivate students' spatial imagination. In addition, we also advance the content that hyperbolic paraboloid is a ruled surface, and this hyperbolic paraboloid can be dynamically generated with a straight line, in order to improve teaching effect of teachers.

2.Equations and graphs of hyperbolic paraboloids

The standard equation of hyperbolic paraboloid in space rectangular coordinate system is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z(a, b > 0)$. According to the equation, the surface is symmetrical about xOz, yOz planes and z-axis, the intersection with the triaxial is (0,0,0), and the surface is unbounded[5].

Through the equations and properties of surface, parallel cutting method is used to analyze shape of surface, that is, three coordinate planes and a plane parallel to the coordinate plane are used to cut the curved surface.

Firstly, three coordinate planes are used to cut the surface. Substituting the equations z=0, x=0 and y=0 of xOy, yOz and zOx planes into the standard equations of the curved surface, equations (1), (2) and (3) are obtained respectively.

$$\begin{cases} \frac{x}{a} \pm \frac{y}{b} = 0 \\ z = 0 \end{cases} (1) \qquad \begin{cases} y^2 = -2b^2z \\ x = 0 \end{cases} (2) \qquad \begin{cases} x^2 = 2a^2z \\ y = 0 \end{cases} \tag{3}$$

Equation (1) is two intersecting lines, equation (2) is a parabola with the opening in the negative direction of z-axis, its vertex is the origin and axis of symmetry is z-axis, equation (3) is still a parabola, which has the same vertex and axis of symmetry as equation (2), but the opening direction is opposite. Equation (2) (3) is called the main parabola.

Secondly, Cutting the surface with a plane parallel to the coordinate plane. If the surface is cut with a plane z=h (h≠0) parallel to xOy plane, the truncation equation is obtained as equation (4), which always represents hyperbola.

$$\begin{cases} \frac{x^2}{2a^2h} - \frac{y^2}{2\,b^2h} = 1 \\ z = h \end{cases} \tag{4}$$

When $h > 0$, equation (4) represents a hyperbola with real axis parallel to x-axis and imaginary axis parallel to y-axis, and the vertex coordinates are ($\pm a\sqrt{2h}$,0,h), which satisfy equation (3), then the vertex is on the main parabola (3). When $h < 0$, equation (4) represents a hyperbola with real axis parallel to y-axis and imaginary axis parallel to x-axis, and the vertex coordinates are (0, $\pm b\sqrt{-2h}$,h), satisfying equation (2), then the coordinates are on the main parabola (2).

If the surface is cut with a plane y=h(h≠ 0) parallel to zOx plane, the cut equation is obtained as equation (5).

$$\begin{cases} \frac{x^2}{a^2} = 2z + \frac{h^2}{b^2} \\ y = h \end{cases} \tag{5}$$

In this equation, whether h>0 or h<0, it represents a parabola with the opening in the positive direction of z-axis, and the vertex coordinate is (0, h, $-\frac{h^2}{2b^2}$), which satisfies equation (2), this coordinate is on the main parabola (2).

To sum up, this surface is divided into upper and lower parts by xOy plane. The upper part rises in two directions of x-axis and the lower part falls in two directions of y-axis. According to analysis of the shape of hyperbolic paraboloid, the approximate figure of hyperbolic paraboloid can be obtained, as shown in Figure 1.
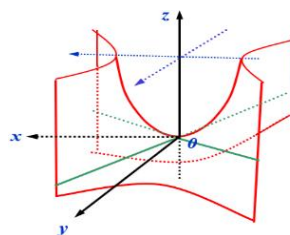

Figure 1. Static hyperbolic paraboloid

3.Application of MATLAB in dynamic paraboloid Teaching

This section builds on the theory of the previous section, the formation process of hyperbolic paraboloid is displayed by Matlab software, and the change of truncation is displayed by parallel cutting method.

3.1 Formation process of hyperbolic paraboloid

Whether h>0 or h<0, the vertices of equation (5) always move on the main transversal represented by equation (2). The transversals represented by both equations are paraboloids with identical vertices and axes but opposite openings. When the parabola of equation (5) is parallel to the plane on which it is located, and the vertex slides on the main section (2), the motion trajectory generated by the parabola is a hyperbolic paraboloid. Set a=3, b=4, the dynamic formation process of hyperbolic paraboloid is shown in Figure 2.
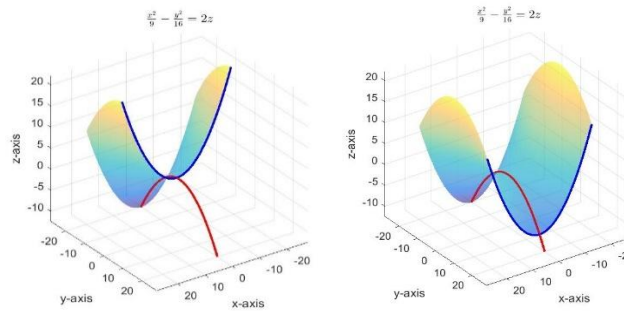

Figure 2. Formation process of hyperbolic paraboloid

Among them, the red parabola in Figure 2 is the main transversal corresponding to equation (2), and the blue parabola corresponds to the main transversal corresponding to equation (5). It can be seen that the generation of hyperbolic paraboloid is generated by one parabola sliding on another parabola perpendicular to it.

3.2 The change process of surface is analyzed by parallel cutting method

According to the idea of parallel cutting method in Section II,this surface is cut with three coordinate planes and planes parallel to coordinate plane. The visualization function of Matlab software is used and the graphic rotation is set to display the change of cutting mark dynamically [6]. The following shows the dynamic demonstration process of cutting hyperbolic paraboloid with parallel cutting method, where a=3, b=4 in the standard equation of hyperbolic paraboloid is set, the running result is shown in figure 3, in which only 4 frames in the process are selected.

(a)   （b）

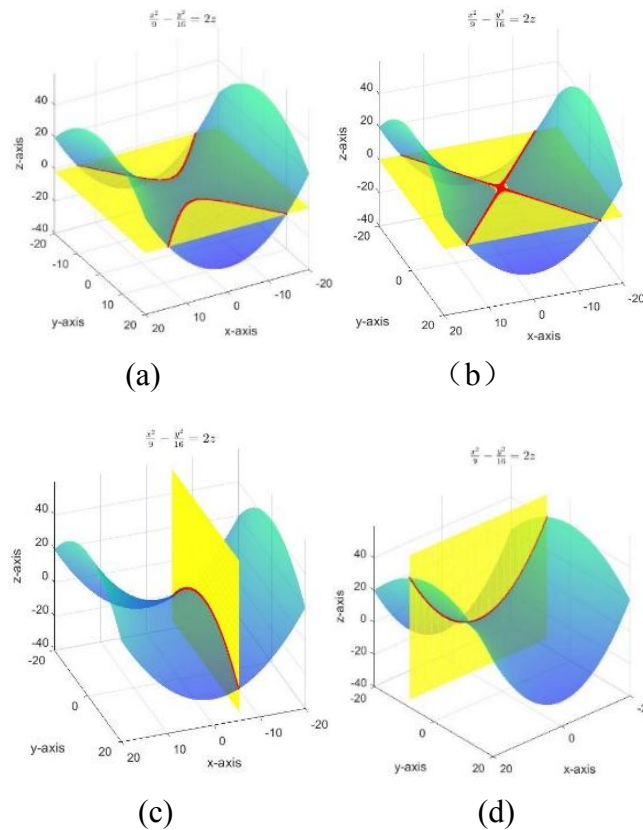(c)   (d)

Figure 3. Dynamic cutting diagram of hyperbolic paraboloid

The red curve in the figure 3 is the intersection of the plane and the curved surface. Figure (a) and (b) are different transversals obtained by cutting with the plane z=h, figure (a) corresponds to the intercept of h>0 in equation (4), that is, the hyperbola with real axis parallel to x-axis and virtual axis parallel to y-axis. Figure (b) corresponds to the graph of equation (1), which is two intersecting lines. Figure (c) is a parabola obtained by cutting with the plane x=h, and its opening is in the negative direction of z-axis. Figure (d) is a parabola obtained by cutting the plane y=h, and its opening is in the positive direction of z-axis. This process not only shows the changing process of analyzing surface with the parallel cutting method, but also shows the 360° all-round graphics, which intuitively and vividly displays the dynamic graphics of the line cutting surface.It is a part of codes that this surface is cut by z = h plane, MakeGif is the M file that saves the dynamic graph.

```
>>clc, close all, clear; nn=360;esp=10e-2; % error
t=linspace(-20,20,nn);[x,y]=meshgrid(t);z=1/18*x.^2-1/32*y.^2;
xlabel(' x-axis '),ylabel(' y-axis '),zlabel(' z-axis ');
title('$\frac{x^2}{9}-\frac{y^2}{16}=2z$','Interpreter','latex')
n=0; %z=h
for i = linspace(min(min(z)),max(max(z)),nn)
 n=n+1; cla; surf(x,y,z),shading interp;
alpha(0.6) % set transparency
 hold on
 z1=i*ones(length(t));h2=mesh(x,y,z1);shading interp
 colormap(winter);set(h2,'edgecolor','y','edgealpha',0.4); hidden off
 a=find(abs(z1-z)<esp);
 plot3(x(a),y(a),z(a),'r.','markerfacecolor','r','markersize',5)
axis vis3d;view(gca,30+n,30)     %set viewing angle
drawnow; MakeGif(' hyperbolic paraboloid.gif',i)
end
```

3.3 The straight line generation process of hyperbolic paraboloid

In teaching, we usually learn a ruled surface generated by a straight line after learning the common quadric surface. In order for students to better master several generation processes of hyperbolic paraboloids, we will advance the relevant contents of the straightness of hyperbolic paraboloids and compare them with the previous generation process. In this section, through the theoretical analysis of straight generatrix, with the help of Matlab software, two families of straight lines are used to generate hyperbolic paraboloids. The u-family and v-family straight generatrix equations corresponding to the standard equation are as follows:

$$\begin{cases} \frac{x}{a}+\frac{y}{b}=2\,u \\ u\left(\frac{x}{a}-\frac{y}{b}\right)=z \end{cases} \text{ and } \begin{cases} \frac{x}{a}-\frac{y}{b}=2\,v \\ v\left(\frac{x}{a}+\frac{y}{b}\right)=z \end{cases} \qquad （6）$$

If the u value is fixed, the simplified parameter equations of the rectangular hyperbolic paraboloid are as follows.

$$\begin{cases} x=x \\ y=b\left(2u-\frac{x}{a}\right) \\ z=u\left(\frac{x}{a}-\frac{y}{b}\right) \end{cases} \text{ and } \begin{cases} x=x \\ y=b\left(\frac{x}{a}-2v\right) \\ z=v\left(\frac{x}{a}+\frac{y}{b}\right) \end{cases} \qquad （7）$$

With the change of u value and v value, two families of straight generatrix will be drawn as ruled surface. The parameter equation is programmed into Matlab, referring to the reference [7], we set a=3, b=4, the variation interval of u is [-20, 20], and take the appropriate variation ranges abuot x, y and z , we will obtain the hyperbolic paraboloid drawn by u-family rectilinear generators, as shown in figure 4 (a). Similarly, the hyperbolic paraboloid generated by v-family rectilinear generators can be obtained, we superimpose two families rectilinear generators into the same figure, as shown in figure 4 (b).
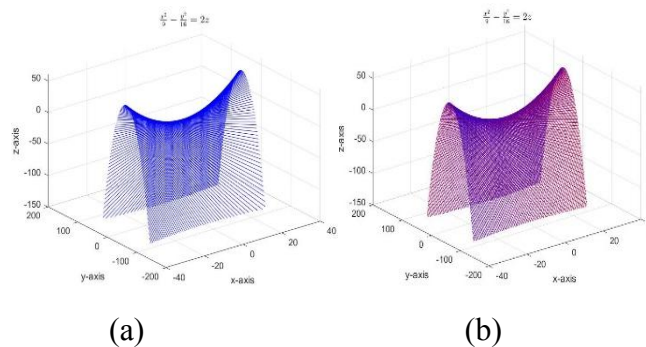

(a)   (b)

Figure4. Generating diagram of two families of straight generatrix of hyperbolic paraboloid

4.Conclusion

In teaching of analytic geometry, Matlab software can usually be used to assist our teaching. Using Matlab software to produce spatial graphics and animation, not only intuitive, but also time saving. In this paper, we show several generation processes of hyperbolic paraboloid, also dissect how the truncation changes in depth. In addition, a hyperbolic paraboloid can also be generated by line families.

On the one hand, the use of dynamic graphics can make abstract problems visualized through graphics visualization, deepen students' understanding of geometric graphics, improve the learning interest of the course, and cultivate students' spatial imagination through various changes of graphics. On the other hand, through active classrooms, the communication and interaction between

teachers and students is enhanced, which greatly improves the teaching effect of teacher, thereby further promoting the modernization process of basic course teaching methods.

Acknowledgment

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