A DPD Algorithm of Maximum Likelihood Based on DOAs and Doppler

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Abstract. Direct position determination (DPD) is a single step method that estimates the position of target by received signals. Localization in 3D scene is a complicated problem, because the signal propagation channel is complex, and it has many kinds of interference and noise. This paper establishes the Over-the-horizon (OTH) shortwave signals and line-of-sight propagation of ultra short wave model, and proposes a joint localization algorithm. We compare the Cramér-Rao Bound(CRB) and the root mean square error (RMSE) between the proposed algorithm and DPD used single information like DOA and Doppler. The simulation results verify the superiority of the proposed algorithm, and prove it has higher accuracy.

Keywords: Joint estimation; direct position determination (DPD); maximum likelihood (ML);direction of arrival (DOA); Doppler.

1. Introduction

Passive localization has always been a hot topic in signal processing, and many scientists have done a lot of research and achievement on passive localization. The most commonly used signals for positioning are shortwave signals and ultrashort wave signals. Traditional method is two-step calculation, such as intersection positioning based on Direction of Arrivals(DOAs)[1], positioning based on Time Difference of Arrival(TDOA) estimation[2] and positioning based on Doppler shift[3]. Wang studies the joint localization based on DOA and TDOA[4].

Amar and Weiss proposed a signal-level information processing method named Direct Position Determination (DPD) [5], which use single step to localization, and improve the accuracy of positioning. Demissie proposed a DPD algorithm based on subspace data fusion (SDF)[6], and then Oispuu proposed Capon-type and deterministic maximum likelihood(DML) DPDs for multiple transmitters[7].

In this paper, we propose a DPD algorithm for 3D scene based on DOAs and Doppler. According to the different propagation models of shortwave signal and ultrashort wave signal, we establish different propagation models and formulas. Finally, we carry out some simulations to verify the performance of the proposed algorithm. By comparing the root mean square error (RMSE) and the Cramér-Rao Bound(CRB) of different algorithms, we find that joint localization has better accuracy.

2. Problem Formulation

2.1 Localization Model

Consider a signal emitter on the Earth, which is transmitting shortwave and ultrashort wave signals $s_1(t)$ and $s_2(t)$ continuously. The latitude and longitude of the emitter is (ζ_r, χ_r) . Assume that there are K_1 shortwave observers and a moving ultrashort wave observer, the latitude and longitude of the k_1th shortwave observer is $(\zeta_{d,k_1}, \chi_{d,k_1}),(1 \le k_1 \le K_1)$. The ultrashort wave observer collects batches of data in K_2 time slots, and the k_2th time slot latitude and longitude of the ultrashort wave observer is $(\zeta_{d,k_2}, \chi_{d,k_2}),(1 \le k_2 \le K_2)$. The shortwave signal propagates beyond the horizon through ionospheric reflection, and the ultrashort wave signal propagates at line of sight. From this we receive the signal propagation model in Fig. 1

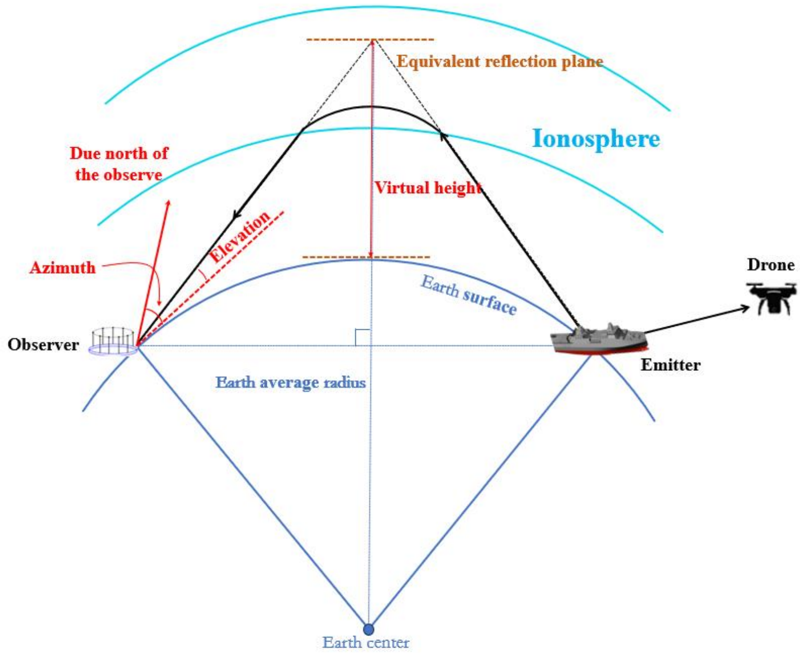

Fig. 1 Signal propagation model

For convenience of calculation, we use the Earth-Centered Earth-Fixed (ECEF) coordinates and assume that the ECEF of the emitter, the shortwave observers, and the ultrashort wave observer are

$$\boldsymbol{u}_r = \left[x_r, y_r, z_r\right]^T, \tag{1}$$

$$\boldsymbol{u}_{d,k_1} = \left[x_{d,k_1}, y_{d,k_1}, z_{d,k_1}\right]^T, (1 \le k_1 \le K_1), \tag{2}$$

$$\boldsymbol{u}_{f,k_2} = \left[x_{f,k_2}, y_{f,k_2}, z_{f,k_2}\right]^T, (1 \le k_2 \le K_2). \tag{3}$$

The calculation formula is as follows.

$$\boldsymbol{u}_r = \frac{1}{\sqrt{1-e^2\sin^2\chi_r}}\begin{bmatrix} r_a\cos(\chi_r)\cos(\zeta_r) \\ r_a\cos(\chi_r)\sin(\zeta_r) \\ r_a(1-e^2)\sin(\chi_r) \end{bmatrix} \tag{4}$$

$$\boldsymbol{u}_{d,k_1} = \frac{1}{\sqrt{1-e^2\sin^2\chi_{d,k_1}}}\begin{bmatrix} r_a\cos(\chi_{d,k_1})\cos(\zeta_{d,k_1}) \\ r_a\cos(\chi_{d,k_1})\sin(\zeta_{d,k_1}) \\ r_a(1-e^2)\sin(\chi_{d,k_1}) \end{bmatrix} \tag{5}$$

$$\boldsymbol{u}_{f,k_2} = \frac{1}{\sqrt{1-e^2\sin^2\chi_{f,k_2}}}\begin{bmatrix} r_a\cos(\chi_{f,k_2})\cos(\zeta_{f,k_2}) \\ r_a\cos(\chi_{f,k_2})\sin(\zeta_{f,k_2}) \\ r_a(1-e^2)\sin(\chi_{f,k_2}) \end{bmatrix} \tag{6}$$

In the formulations above, $e = \frac{\sqrt{r_a^2 - r_b^2}}{r_a} = 0.0818198$, and $r_a = 6378.137\text{km}$ means the major axis of the Earth reference ellipsoid, and $r_b = 6371.393\text{km}$ means the minor axis of the Earth reference ellipsoid.

Assume that the ionospheric virtual height at which the shortwave signal reaches the k_1th observer is h_{k_1}, the azimuths (clockwise location from true north) and the elevation angles of shortwave observers are θ_{k_1} and ψ_{k_1}, expressed as

$$\theta_{k_1} = \arctan\left(\frac{x_r - x_{d,k_1}}{y_r - y_{d,k_1}}\right) \quad (k_1 = 1, 2, \ldots, K_1) \tag{7}$$

$$\psi_{k_1} = \arctan\left(\cot(\beta_{k_1}) - \frac{r_b}{r_b + h_{k_1}}\csc(\beta_{k_1})\right) = \arctan\left(\frac{\left(r_b + h_{k_1}\right)\cos(\beta_{k_1}) - r_b}{\left(r_b + h_{k_1}\right)\sin(\beta_{k_1})}\right) \quad (k_1 = 1, 2, \ldots, K_1) \tag{8}$$

In equation (8), $\beta_{k_1} = \arcsin\left(\frac{1}{2r_b}\left\|\mathbf{u}_r - \mathbf{u}_{d,k_1}\right\|_2\right)(k_1 = 1, 2, \ldots, K_1)$. Assuming that the length of each time slot is significantly shorter than the time interval between two time slots, the spatial array response and the Doppler frequency shift remain constant during each time slot. The signal vector of the k_1th shortwave observer is

$$\boldsymbol{x}_{k_1}(\boldsymbol{t}) = [\boldsymbol{x}_{k_1}(t_1), \boldsymbol{x}_{k_1}(t_2), \ldots, \boldsymbol{x}_{k_1}(t_N)] = \boldsymbol{a}_{k_1}(\theta_{k_1}, \psi_{k_1})\boldsymbol{s}_{k_1} + \boldsymbol{n}_{k_1} \quad (k_1 = 1, 2, \ldots, K_1), \tag{9}$$

Where t_n means the time of n_{th} snapshot, and N represents the snapshot number, $\boldsymbol{a}_{k_1}(\theta_{k_1}, \psi_{k_1})$ is the array epidemic vector of the emitter to the k_1th shortwave observer, $\boldsymbol{s}_{k_1} = [s_{k_1}(t_1), s_{k_1}(t_2), \ldots, s_{k_1}(t_N)]$ is the vector of the complex envelope of a shortwave signal. $\boldsymbol{n}_{k_1}$ is the additive white Gaussian noise in the signal receiving process at the k_{th} observer. The received signal is sampled at $t = nT_s\left(n = 1, 2, \ldots, N\right)$, and we can receive

$$\boldsymbol{x}_{k_1}\left(n\right) = \boldsymbol{a}_{k_1}\left(\theta_{k_1}, \psi_{k_1}\right)\boldsymbol{s}_{k_1}\left(n\right) + \boldsymbol{n}_{k_1}\left(n\right) \quad (n = 1, 2, \ldots, N). \tag{10}$$

Similarly, the signal vector of the ultrashort wave observer in k_2th time slot is

$$\boldsymbol{x}_{k_2}\left(n\right) = \boldsymbol{a}_{k_2}\left(\theta_{k_2}\right)\boldsymbol{s}_{k_2}\left(n\right)e^{j2\pi f_{k_2}(\boldsymbol{u}_r)nT_S} + \boldsymbol{n}_{k_2}\left(n\right)(n = 1, 2, \ldots, N, k_2 = 1, 2, \ldots, K_2) \tag{11}$$

$\boldsymbol{a}_{k_2}\left(\theta_{k_2}\right)$ is the array epidemic vector of the emitter to the ultrashort wave observer in k_2th time slot, The formula for θ_{k_2} is

$$\theta_{k_2} = \arccos\left(\frac{x_r - x_{f,k_2}}{\left\|x_r - x_{f,k_2}, y_r - y_{f,k_2}, z_r - z_{f,k_2}\right\|_2^2}\right) \quad (k_2 = 1, 2, \ldots, K_2), \tag{12}$$

$\boldsymbol{s}_{k_2} = [s_{k_2}(t_1), s_{k_2}(t_2), \ldots, s_{k_2}(t_N)]$ is the vector of the complex envelope of the ultrashort wave signal in k_2th time slot. $f_{k_2}\left(\boldsymbol{u}_r\right)$ is the Doppler shift generated by the received signal when the ultrashort wave observer is moving, which is expressed as

$$f_{k_2}\left(\boldsymbol{u}_r\right) = f_c \frac{\boldsymbol{v}_{k_2}^{\mathrm{T}}\left(\boldsymbol{u}_r - \boldsymbol{u}_{k_2}\right)}{c\left\|\boldsymbol{u}_r - \boldsymbol{u}_{k_2}\right\|_2}. \tag{13}$$

f_c is the carrier frequency of ultrashort wave signal, $\boldsymbol{v}_{k_2} = \left[\mathrm{v}_{\mathrm{x,k_2}} \quad \mathrm{v}_{\mathrm{y,k_2}} \quad \mathrm{v}_{\mathrm{z,k_2}}\right]^{\mathrm{T}}$ is the velocity of ultrashort wave observer, $\boldsymbol{n}_{k_2}$ is the additive white Gaussian noise in the signal receiving process in the k_2th time slot.

2.2 Proposed algrithm

2.2.1 Cost function of shortwave signal

Because of the gaussian noise, we can write the likelihood function of $\boldsymbol{x}_{k_1}$, which is

$$p\left(\boldsymbol{x}_{k_1}|\boldsymbol{\eta}_1\right)=\frac{1}{\left(\pi\sigma_{k_1}^2\right)^{M_{k_1}K_1N}}\cdot\exp\left\{-\frac{1}{\sigma_{k_1}^2}\sum_{k_1}^{K_1}\left\|\boldsymbol{x}_{k_1}-\boldsymbol{a}_{k_1}(\theta_{k_1},\psi_{k_1})\boldsymbol{s}_{k_1}\right\|_2^2\right\}(k_1=1,2,\ldots,K_1), \tag{14}$$

where $\boldsymbol{\eta}_1=\left[\boldsymbol{u}_r,\boldsymbol{s}_{k_1}\right]$. The log-ML-based function is expressed as

$$L(\boldsymbol{\eta}_1)=-M_{k_1}K_1N\ln\left(\pi\sigma_{k_1}^2\right)-\frac{1}{\sigma_{k_1}^2}\sum_{k_1}^{K_1}\left\|\boldsymbol{x}_{k_1}-\boldsymbol{a}_{k_1}(\theta_{k_1},\psi_{k_1})\boldsymbol{s}_{k_1}\right\|_2^2(k_1=1,2,\ldots,K_1). \tag{15}$$

After omitting the constant term, we can get the cost function of received shortwave signal model, which is expressed as

$$V_1\left(\boldsymbol{u}_r,\boldsymbol{s}_{k_1}\right)=\sum_{k_1}^{K_1}\left\|\boldsymbol{x}_{k_1}-\boldsymbol{a}_{k_1}(\theta_{k_1},\psi_{k_1})\boldsymbol{s}_{k_1}\right\|_2^2,(k_1=1,2,\ldots,K_1). \tag{16}$$

We can calculate the estimation of $\boldsymbol{s}_{k_1}$ by the least squares estimate:

$$\hat{\boldsymbol{s}}_{k_1}=\left[\boldsymbol{a}_{k_1}^{\mathrm{H}}(\theta,\psi_{k_1})\boldsymbol{a}_{k_1}(\theta,\psi_{k_1})\right]^{-1}\boldsymbol{a}_{k_1}^{\mathrm{H}}(\theta,\psi_{k_1})\boldsymbol{x}_{k_1} \tag{17}$$

Substitute $\hat{\boldsymbol{s}}_{k_1}$ into equation (15), the cost function can be written as

$$V_1\left(\boldsymbol{u}_r\right)=\sum_{k_1}^{K_1}\left\|\boldsymbol{\Pi}_{k_1}^{\perp}\left(\boldsymbol{a}_{k_1}(\theta,\psi_{k_1})\right)\boldsymbol{x}_{k_1}\right\|_2^2, \tag{18}$$

where $\boldsymbol{\Pi}_{k_1}^{\perp}\left(\boldsymbol{a}_{k_1}(\theta_{k_1},\psi_{k_1})\right)=\boldsymbol{I}_{M_{k_1}}-\boldsymbol{a}_{k_1}^{\dagger}(\theta_{k_1},\psi_{k_1})\boldsymbol{a}_{k_1}(\theta_{k_1},\psi_{k_1})$ is the orthogonal projection matrix of the $\boldsymbol{a}_{k_1}(\theta_{k_1},\psi_{k_1})$ complement, M_{k_1} is the order of $\boldsymbol{a}_{k_1}(\theta_{k_1},\psi_{k_1})$.

2.2.2 Cost function of ultrashort wave signal

The likelihood function of $\boldsymbol{x}_{k_2}$ is

$$p\left(\boldsymbol{x}_{k_2}|\boldsymbol{\eta}_2\right)=\frac{1}{\left(\pi\sigma_{k_2}^2\right)^{M_{k_2}K_2N}}\cdot\exp\left\{-\frac{1}{\sigma_{k_2}^2}\sum_{k_2}^{K_2}\left\|\boldsymbol{x}_{k_2}-\boldsymbol{a}_{k_2}(\theta_{k_2})\boldsymbol{s}_{k_2}\right\|_2^2\right\}(k_2=1,2,\ldots,K_2), \tag{19}$$

where $\boldsymbol{\eta}_2=\left[\boldsymbol{u}_r,\boldsymbol{s}_{k_2}\right]$. The log-ML-based function is expressed as

$$L_2\left(\boldsymbol{u}\right)=-M_{k_2}LK_2N\ln\left(\pi\sigma_{k_2}^2\right)-\frac{1}{\sigma_{k_2}^2}\sum_{k_2}^{K_2}\left\|\boldsymbol{x}_{k_2}-\boldsymbol{a}_{k_2}(\theta_{k_2})\boldsymbol{s}_{k_2}e^{j2\pi f_{k_2}(\boldsymbol{u})nT_S}\right\|_2^2(k_2=1,2,\ldots,K_2). \tag{20}$$

For convenience of calculation, write the ultrashort wave signal model as

$$\boldsymbol{x}_{k_2}^{\mathrm{T}}(n)=\boldsymbol{b}_{k_2}\left(\theta_{k_2},f_{k_2}\left(\boldsymbol{u}_r\right)\right)\boldsymbol{s}_{k_2}^{\mathrm{T}}(n)+\boldsymbol{n}_{k_2}^{\mathrm{T}}(n)(n=1,2,\ldots,N,k_2=1,2,\ldots,K_2), \tag{21}$$

where $\boldsymbol{b}_{k_2}\left(\theta_{k_2},f_{k_2}\left(\boldsymbol{u}_r\right)\right)=\boldsymbol{F}_{k_2,2}\left(\boldsymbol{u}_r\right)\otimes\boldsymbol{a}_{k_2}(\theta_{k_2})$,

$$\boldsymbol{F}_{k_2,2}\left(\boldsymbol{u}_r\right)=\mathrm{diag}\left(\left[e^{j2\pi f_{k_2}(\boldsymbol{u}_r)T_s},e^{j4\pi f_{k_2}(\boldsymbol{u}_r)T_s},\ldots,e^{j2N\pi f_{k_2}(\boldsymbol{u}_r)T_s}\right]^{\mathrm{T}}\right) \tag{22}$$

The cost function of received ultrashort wave signal model is

$$V_2\left(\boldsymbol{u},\boldsymbol{s}_{k_2}\right)=\sum_{k_2}^{K_2}\left\|\boldsymbol{x}_{k_2}^{\mathrm{T}}-\boldsymbol{b}_{k_2}\left(\theta_{k_2},f_{k_2}\left(\boldsymbol{u}\right)\right)\boldsymbol{s}_{k_2}^{\mathrm{T}}\right\|_2^2,(k_2=1,2,\ldots,K_2). \tag{23}$$

We can calculate the estimation of $\boldsymbol{s}_{k_2}$ by the least squares estimate:

$$\hat{\boldsymbol{s}}_{k_2}^{\mathrm{T}}=\left[\boldsymbol{b}_{k_2}^{\mathrm{H}}\left(\theta_{k_2},f_{k_2}\left(\boldsymbol{u}\right)\right)\boldsymbol{b}_{k_2}\left(\theta_{k_2},f_{k_2}\left(\boldsymbol{u}\right)\right)\right]^{-1}\boldsymbol{b}_{k_2}^{\mathrm{H}}\left(\theta_{k_2},f_{k_2}\left(\boldsymbol{u}\right)\right)\boldsymbol{x}_{k_1}^{\mathrm{T}}. \tag{24}$$

Substitute $\hat{\boldsymbol{s}}_{k_2}$ into equation (22), the cost function can be written as

$$V_2\left(\boldsymbol{u}\right)=\sum_{k_2}^{K_2}\left\|\boldsymbol{\Pi}_{k_2}^{\perp}\left(\boldsymbol{b}_{k_2}^{\mathrm{H}}\left(\theta_{k_2},f_{k_2}\left(\boldsymbol{u}\right)\right)\right)\boldsymbol{x}_{k_2}\right\|_2^2,(k_2=1,2,\ldots,K_2). \tag{25}$$

2.2.3 Joint localization method

We can obtain the final cost function by a weighted sum of V_1 and V_2, which is expressed as

$$V\left(\boldsymbol{u}\right)=W_1V_1\left(\boldsymbol{u}\right)+W_2V_2\left(\boldsymbol{u}\right). \tag{26}$$

W_1 and W_2 can be obtained by the covariance matrixs of $\boldsymbol{x}_{k_1}$ and $\boldsymbol{x}_{k_2}$,which is expressed as

$$\boldsymbol{R}_{K_1}=\boldsymbol{x}_{k_1}\boldsymbol{x}_{k_1}^{\mathrm{H}}, \tag{27}$$

$$\boldsymbol{R}_{K_2}=\boldsymbol{x}_{k_2}\boldsymbol{x}_{k_2}^{\mathrm{H}}. \tag{28}$$

We can decompose $\boldsymbol{R}_{K_1}$ by eigenvalues and get M_{k_1} eigenvalues, then average the eigenvalues of the smallest $M_{k_1}-1$, finally get the estimation of $\sigma_{k_1}^2$,which is expressed as

$$\hat{\sigma}_{k_1}^2=\frac{1}{M_{k_1}-1}\mathrm{sum}\left(eig_2\left(\boldsymbol{R}_{K_1}\right),eig_3\left(\boldsymbol{R}_{K_1}\right),\ldots,eig_{M_{k_1}}\left(\boldsymbol{R}_{K_1}\right)\right). \tag{29}$$

W_1 is written as $\frac{1}{\hat{\sigma}_{k_1}^2}$. Similarly, W_2 is written as $\frac{1}{\hat{\sigma}_{k_2}^2}$,

$$\hat{\sigma}_{k_2}^2=\frac{1}{M_{k_2}-1}\mathrm{sum}\left(eig_2\left(\boldsymbol{R}_{K_2}\right),eig_3\left(\boldsymbol{R}_{K_2}\right),\ldots,eig_{M_{k_2}}\left(\boldsymbol{R}_{K_2}\right)\right). \tag{30}$$

Finally, we can use the grid search method to find the estimated coordinates of the emitter， the result is expressed as

$$\hat{\boldsymbol{u}}_{r,\mathrm{ML}}=\mathrm{argmin}\left\{V\left(\boldsymbol{u}\right)\right\}. \tag{31}$$

3. Simulation and Analysis

In this section, we carry out some simulations to check the performance of the proposed algorithm. We use 3 shortwave observers, each of which has uniform circular arrays with $M_{k_1}=10$. And the ultrashort wave moving observer has uniform linear array with $M_{k_2}=3$. The velocity vector of the ultrashort wave observer is $\boldsymbol{v}_{k_2}=\begin{bmatrix}0.1 & 0.2 & 0.2\end{bmatrix}$km/s . Their latitude and longitude coordinates are in the Table 1. Fig. 2 shows their location.

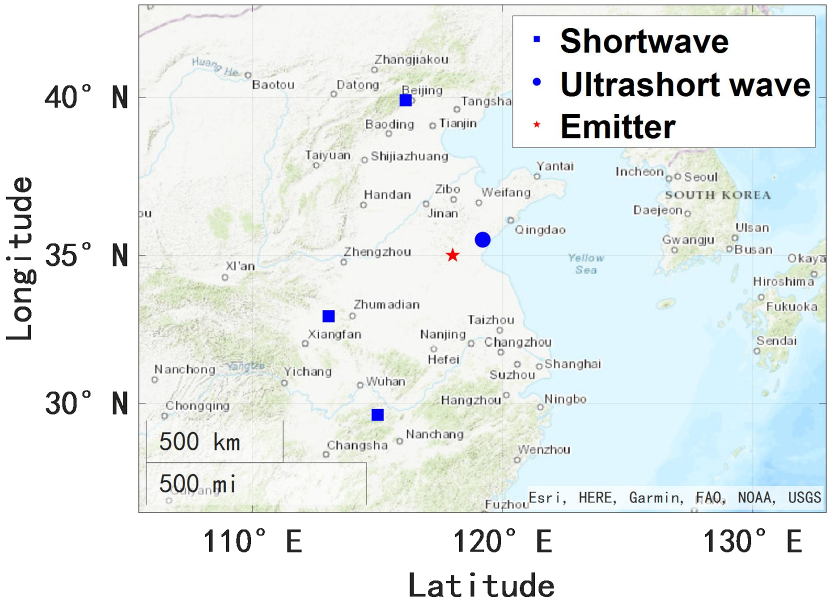

Fig 2 Location of emitter and observers

Table 1. Latitude and longitude coordinates of emitter and observers

	Emitter	Shortwave 1	Shortwave 2	Shortwave 3	Ultrashort wave
Latitude/°	118.00°E	116.12°E	113.04°E	115.00°E	119.20°E
longitude/°	35.00°N	39.92°N	32.98°N	29.61°N	36.51°N
Ionospheric height/km	\	340	360	375	\

We use the RMSE of localization to represent the performance of the algorithm:

$$RMSE = \frac{1}{num}\sqrt{\sum_{i=1}^{num}\left\|\boldsymbol{u}_r - \hat{\boldsymbol{u}}_{r,i}\right\|_2^2}, \tag{32}$$

where num means the times of Monte Carlo experiments, and num is set to 100. The Cramér-Rao Bound(CRB) provides a benchmark for the highest localization accuracy achievable for any unbiased estimator.

Firstly, we verify the effectiveness of joint localization by comparing the CRB between the algorithms, Fig. 3 shows the proposed algorithms has lower CRB, which proved that combining multiple information can improve the performance limit of localization.

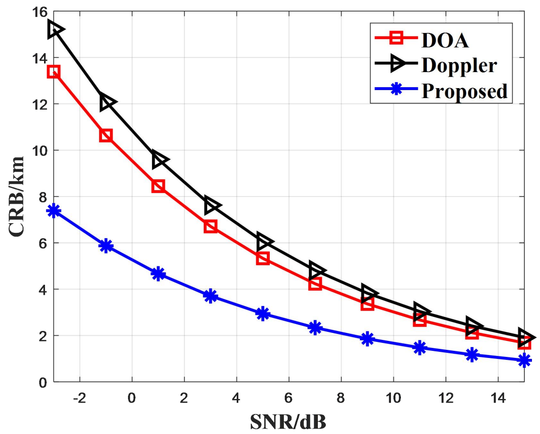

Fig. 3 Comparison of CRB versus SNR

Then, we compare the RMSE of each algorithms, and Fig. 4 shows the results. From Fig. 4, we can find that joint localization has better accuracy.

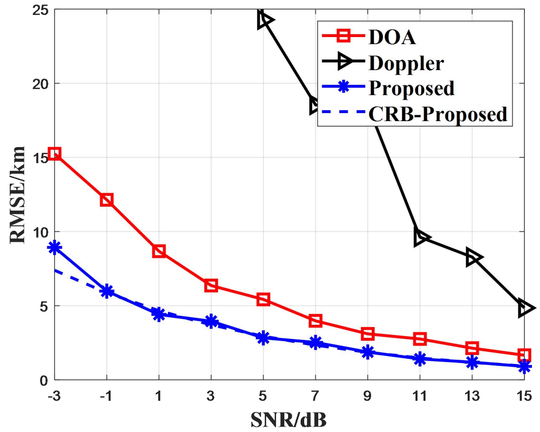

Fig. 4 Comparison of RMSE versus SNR

4. Summary

This paper studies the problem of passive localization, and modeled the joint information localization in 3D scene. According to the propagation characteristics of shortwave signal and ultrashort wave signal, we calculate their DOAs information, Doppler information. Limited by the length of the article, the calculation formula of CRB is not derived in detail. We prove that the combination of multiple information can improve the positioning accuracy, obtain the CRB and RMSE.

References

[1] Guanghui Chen, Xiaoping Zeng, Shuang Jiao. Near-Field DOA Estimation Algorithm Using Array Partition[J]. Acta Electronica Sinica, 2022, 50(9): 2110-2118.

[2] Fan Meng, Shengheng Liu, et al. TDoA positioning with data-driven LoS inference in mmWave MIMO communications[J]. Signal Processing, 2024, 220.

[3] Jiexin Yin, Ding Wang, Ying Wu, et al. Direct Localization of Multiple Stationary Narrowband Sources Based on Angle and Doppler. IEEE Communications Letters, 2017, 21(12): 2630-2633.

[4] Ding Wang, Jiexin Yin, Zhongliang Zhu. Novel cooperative localization method of over-the-horizon shortwave emitters based on direction-of-arrival and time-difference-of-arrival measurements. Sci Sin Inform, 2022, 52: 1942–1973.

[5] Alon. Amar, Anthony. J. Weiss, Direct position determination of multiple radio signals. IEEE International Conference on Acoustics, Speech, and Signal Processing, Montreal, QC, Canada, 2004.

[6] Bruno Demissie, Marc Oispuu, Eick Ruthotto, Localization of multiple sources with a moving array using subspace data fusion. International Conference on Information Fusion, Cologne, Germany, 2008, 1-7.

[7] Marc Oispuu. Direct state determination of multiple sources with intermittent emission. 2009 17th European Signal Processing Conference, Glasgow, UK, 2009, 1948-1952.

[8] Wanting Yu, Hongyi Yu, Jianping Du, Ding Wang, A Direct Position Determination Method for Over-the-Horizon Target on Known Radiation Source Waveforms. Acta Electronica Sinica,2019, 47(11): 2368-2377.

[9] Hang Jiang, Jianfeng Li, Kehui Zhu, Yingying Li. Sparse direct position determination based on TDOA information in correlation-domain. Remote Sens, 2023, 15, 3705.

[10] Ding Wang, Gang Zhang. A direct position determination algorithm for constant modulus signals with single moving observer. Acta Aeronauticaet Astronautica Sinica, 2016, 37(5), 1622-1633.

[11] Ding Wang, Jiexin Yin, Zhidong Wu, Ruirui Liu. Direct localization method for constant modulus source based on Doppler frequency shifts[J]. Acta Aeronauticaet Astronautica Sinica, 2017, 38(9): 321084-321084.