Applicability of Mooney-Rivlin model for studying hyperelastic behavior of tongue

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Abstract. The hypertrophy and relaxation of tongues easily leads to patients' airway obstruction during sleep, which will cause obstructive sleep apnea hypopnea syndrome (OSAHS). Therefore, studying biomechanical properties of tongues with constitutive relation to obtain related parameters is important. In this paper, applicability study is performed to specify the tongues' mechanical properties (TMP) based on the Mooney-Rivlin hyperelastic model. Firstly, the research on uniaxial stretching vitro experiments of pig tongues was carried out; then, the material parameters of the proposed model were determined with reference to the experiments results; finally, the advantages and disadvantages of the model to detail the TMP were discussed respectively. The results showed that the Mooney-Rivlin hyperelastic model has a simple structure, which reflects the TMP well while appearance of large deformation but fails to reflect the TMP while the strain is small in linear segment, but that of the toe segment is in contrast to the linear segment.

Keywords: Mooney-Rivlin model; hyperelastic; tongue; obstructive sleep apnea hypopnea syndrome (OSAHS).

1. Introduction

Obstructive sleep apnea hypopnea syndrome (OSAHS) is the apnea caused by upper airway collapse and obstruction during the patients' sleep, accompanied by hypoventilation, snoring, frequent descent of the oxygen saturation and sleep disorder [1]. Currently, it is believed that three major factors lead to OSAHS disease, including abnormal anatomical structure of upper airway, abnormal dilatator muscle of upper airway and abnormal function of pneumotaxic center, all of which are closely related to the mechanical properties of the soft tissue in upper airway' s cavity [2]. As the tongue mechanical properties change, the movement and deformation of the tongue in the patient's cavity will also change during sleep respiratory process, resulting in airway obstruction [3].

Human tongue usually exhibits the characteristics of large nonlinear deformation, including anisotropic, viscoelastic, inhomogeneity and approximate incompressibility [4]. Similar to the stress-strain curve of rubber (pure elastomer), human tongue can generate large elastic deformation with an elastic limit 95% of the strength limit, which allows them to be stretched almost to the point of rupture without plastic deformation. The mechanical behaviors of soft tissue are time-dependent. Soft tissue is still assumed as hyperelastomers in the range of low strain rate, indicating that the deformation of the hyperelastomer [5] can be fully recovered.

In this paper, based upon the assumption that human tongue is incompressible, the Mooney-Rivlin model for depicting the constitutive relationship of human tongue was proposed. The material parameters were determined with reference to the research on uniaxial stretching vitro experiment of pig tongue. The Mooney-Rivlin model to describe tongue mechanical properties were discussed by comparing with the experimental results.

2. Applicability of hyperelastic Mooney-Rivlin model

2.1 Hyperelastic Mooney-Rivlin model

Large elastic concentrated constitutive theory gleaned from strain energy density has been developed and applied to hyperelastic materials. These constitutive relations can be prevailingly divided into two types: The first one is that strain energy density is considered to be a polynomial function of the main strain invariant. When the material is incompressible, it is commonly referred to as the Rivlin material and as the Mooney-Rivlin material if only one entry is adopted. The second considers that strain energy density is an independent function of three principal elongation ratios, such as the material model of Ogden, Peng and Pneg-Landel [6].

Early as 1940, in order to study rubber-like nonlinear materials, Mooney [7] used one order solely on the basis of the Rivlin model to obtain the hyperelastic Mooney-Rivlin model, as shown in Eq. (1). When the strain of rubber material is less than 150%, the model can nicely describe its mechanical properties.

$$W_{\text{Mooney}} = C_1(I_1 - 3) + C_2(I_2 - 3) \tag{1}$$

where, W is the function of strain energy density; I_1 and I_2 are the invariant of deformation tensor; C_1 and C_2 are the constants of material mechanical properties, which can be obtained by uniaxial tensile test.

The hyperelastic Mooney-Rivlin theory was constructed based upon the two hypotheses:

(1) The material is incompressible and isotropic before deformation.

(2) The shear deformation follows Hooke's law, so does the superimposed shear in planes perpendicular to unidirectional stretching or compression.

2.2 Determine parameters of Mooney-Rivlin model

The strain-energy function W of Mooney-Rivlin is widely used in nonlinear finite element models of elastomers. From the relationship between Kirchoff stress tensor t_{ij} and Green strain tensor γ_{ij}, the following equation can be deduced [8]:

$$t_{ij} = \frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial \gamma_{ij}} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial \gamma_{ij}} + \frac{\partial W}{\partial I_3}\frac{\partial I_3}{\partial \gamma_{ij}} \tag{2}$$

Since hypothesized that the material is incompressible, the value of I_3 is 1. The relation between principal stress t_i and principal elongation λ_i can be attained by simplifying Eq.(2), as seen as Eq.(4):

$$t_i = 2(\lambda_i^2 \frac{\partial W}{\partial I_1} + \frac{1}{\lambda_i^2}\frac{\partial W}{\partial I_2}) \tag{3}$$

Difference of three principal stresses can be gained by Eq. (3):

$$\begin{aligned} t_1 - t_2 &= 2(\lambda_1^2 - \lambda_2^2)(\frac{\partial W}{\partial I_1} + \lambda_3^2 \frac{\partial W}{\partial I_2}) \\ t_2 - t_3 &= 2(\lambda_2^2 - \lambda_3^2)(\frac{\partial W}{\partial I_1} + \lambda_1^2 \frac{\partial W}{\partial I_2}) \\ t_3 - t_1 &= 2(\lambda_3^2 - \lambda_1^2)(\frac{\partial W}{\partial I_1} + \lambda_2^2 \frac{\partial W}{\partial I_2}) \end{aligned} \tag{4}$$

It is supposed that the strain energy density is a polynomial function of the principal strain invariants, and the strain energy function should be symmetric with respect to the three principal elongation ratios under the assumption of isotropy. Therefore, λ_i=1 +ε_i (i=1,2,3)， and ε_i is the strain of materials.

In this paper, the material parameters of the Mooney-Rivlin model were determined by the vitro experimental data from the uniaxial stretching of pig tongue. In the condition of uniaxial stretch experiment, t_2=t_3=0, and hence equation λ_2^2=λ_3^2=1/λ_1 was constructed.

Combining Eq. (2) and Eq. (4), the relationship among the principal stress, strain of the Mooney-Rivlin model for incompressible materials and the principal elongation ratio ofdeformation tensor invariants was deduced:

$$t_1 = 2(\lambda_1^2 - \frac{1}{\lambda_1})(C_1 + \frac{1}{\lambda_1}C_2) \tag{5}$$

Here, the main stress t_1 is the true stress, which is transformed into the engineering stress σ, $\sigma=t/\lambda$. The final stress-elongation ratio equation was expressed as:

$$\sigma = 2(\lambda - \frac{1}{\lambda^2})(C_1 + \frac{1}{\lambda}C_2) \tag{6}$$

In this paper, the material mechanical parameters C_1 and C_2 were determined by the results of the uniaxial stretching experiment of pig tongue (transverse direction of tongue apex). The stress-strain curve gained by the experiment has obvious toe segment and linear segment, as shown in Fig.1. Therefore, the corresponding material parameters C_1 and C_2 should be valued by the two segments respectively. The experimental data were redrawn in the coordinate system with $1/\lambda$ as the x-coordinate and $\sigma/[2(\lambda-1/\lambda^2)]$ as the y-coordinate, according to the experimental data in Fig.2 and the relationship of stress-elongation ratio in the Mooney-Rivlin model. The linear regression formula was used to fit the experimental data via MATLAB, setting C_1 as the intercept of the regression line and C_2 as the slope, as diagrammed in Fig.2. The material parameters C_1, C_2 in toe segment of the hyperelastic Mooney-Rivlin constitutive model were 0.744, -0.776, respectively; while C_1, C_2 were 0.311, -0.241 respectively in linear segment.

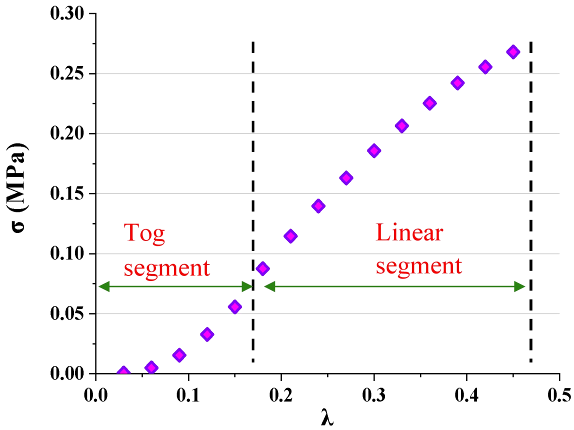
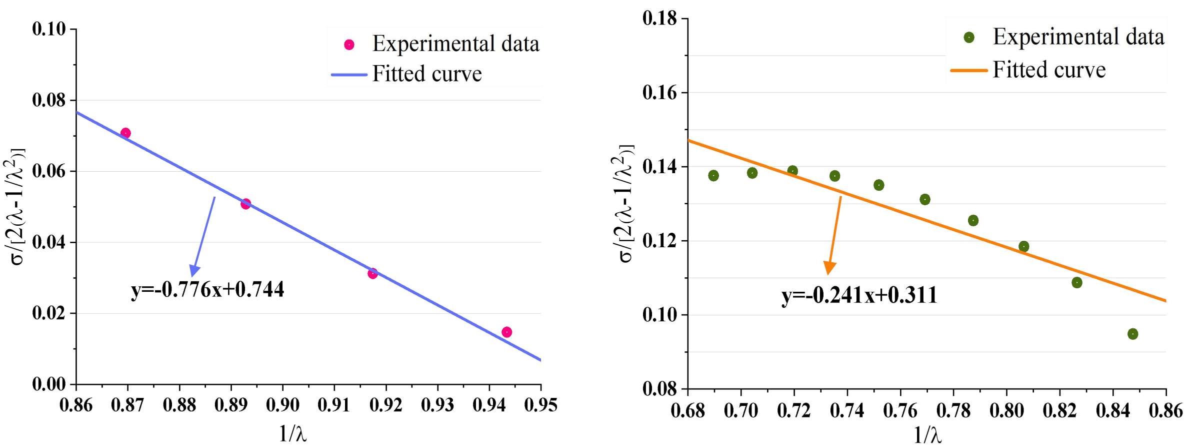


Fig. 1 Experimental data of transverse uniaxial tension of tongue apex



(a) Fitted curve of toe segment (b) Fitted curve of linear segment

Fig. 2 Fitted curves of Mooney-Rivlin model

3. Discussion

The stress-strain experimental data corresponding to the toe segment and linear segment of the Mooney-Rivlin model were drawn in Fig.3. The following conclusions were concluded by comparison:

(1) The stress-strain curves in toe segment and linear segment of the Mooney-Rivlin model are apparently distinct. To generate the same deformation effect, the force required for the toe segment is less than the linear segment, with a two-times gap of force approximately.

(2) The fitting degree of the toe segment is better when the strain is small, but worse when the strain becomes large, while that of the linear segment is in contrast to the toe segment. Consequently, considering segmental definition according to the value range of strain is crucial when the Mooney-Rivlin model is adopted for describing the mechanical behaviour of a certain type of soft tissue.

(3) The crosspoint (ε_1=0.236) of strain-stress curves in tog segment and linear segment can be employed as the segregating point of the simulation analysis. If the strain is less than 0.236, the material parameters of the toe segment are applied in the model, otherwise those of the linear segment are used.

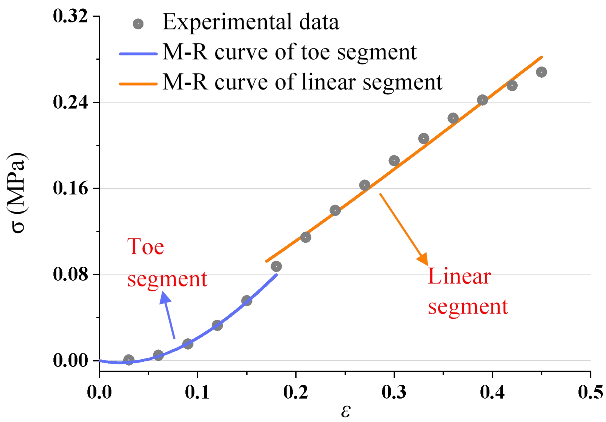


Fig. 3 Comparison of stress and strain between model and experiment

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