Inversion of Ocean Wave Spectrum from the Multi-frequency HF Radar Sea Echoes with Wind and Swell Coexisting

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Abstract. Recently, the multi-frequency HF radar has attracted much attention. However, only a few works focus on the method of obtaining wave information directly. In this work, a wave spectrum inversion method for multi-frequency HF radar in complex sea conditions is proposed. This method considers the swell component in the ocean, and uses multiple HF radars of different frequencies to obtain the echo Doppler spectra from complex sea conditions with wind and swell coexisting. And then the wave spectrum is derived from the high-energy outer second-order spectra and the corresponding first-order peak of the Doppler spectra. In addition, the proposed method is applied to the simulated sea echo data and the result indicates that the method is effective.

Keywords: Multi-frequency; HF radar; Swell; Wave spectrum.

1. Introduction

Compared with the single-frequency HF radar, the multi-frequency HF radar has stronger anti-interference and self-correction capabilities, as well as its ability to provide more information on ocean wave dynamics.

In the past few decades, there have been numerous studies on multi-frequency HF radar. In 1991, Anderson proposed a layered classification multi-frequency inversion technique [1], which was applied to HF skywave radar and HF groundwave radar systems. In 2014, Tian proposed a new dual-frequency wave height measurement method, which combines the results obtained from two frequencies [2]. Recently, Chen et al. established the direct relationship between the multi-frequency echo signal and the wave spectrum, which has certain advantages for wave measurement [3]. However, none of these methods have taken into account the complex sea conditions caused by wind waves and swells.

Different from the previous methods, a wave spectrum inversion method for multi-frequency HF in complex sea conditions is proposed. This method considers the swell component in the ocean, and uses multiple HF of different frequencies to obtain the echo Doppler spectra of mixed waves in complex sea conditions. The ratio of the outer second-order spectrum and the energy of the first-order peak on the stronger side of the spectrum is taken, and multiplied by the pseudo-inverse of the undirected wave spectrum coefficient matrix obtained through principal component analysis to obtain the fused wave spectrum and directly obtain wave parameters.

This work is organized as follows. Section 2 describes the theory of wave spectra inversion for multi-frequency HF radar. The inversion method for wave spectrum with wind and swell coexisting is given in Section 3. The simulation analysis in Section 4. Section 5 presents the summary.

2. Wave Measurement Theory And Inversion Algorithm

2.1 Radar Cross-section Equation

According to perturbation theory, the first-order and second-order scattering equations, in the absence of surface currents and for deep water, can be expressed as follows [4,5].:

$$\sigma^{(1)}(\omega,\varphi)=2^{6}\pi k_0^4\sum_{m=\pm1}S(-2m\vec{k_0})\cdot\delta(\omega-m\omega_B) \tag{1}$$

$$\sigma^{(2)}(\omega)=2^{6}\pi k_0^4\sum_{m,m'=\pm1}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}|\Gamma|^2 S(m\vec{k})\cdot S(m'\vec{k'})\delta(\omega-m\sqrt{gk}-m'\sqrt{gk'})dpdq \tag{2}$$

where ω is the Bragg frequency; k_0 is the radar wavevector (k_0 is the scalar magnitude of $\vec{k_0}$), and $S(\vec{\cdot})$ is the oriented wave spectrum; according to the dispersion relation, the Bragg frequency is represented by the deep-water radar wave number k_0 , where g is the gravitational acceleration; $\delta(\cdot)$ is the Dirac function; the spatial wave number p is defined to be perpendicular to q along the radar beam; $\vec{k}$ and $\vec{k'}$ are the two rows of the wave vectors resulting from the second-order scattering of the radar beam with the waves, where $\vec{k}=(p-k_0,q)$ and $\vec{k'}=(-(p+k_0),-q)$, and which satisfy the following relations with the radar wave vector: $\vec{k}+\vec{k'}=-2\vec{k_0}$; m and m' take the value ±1, which defines four possible combinations of the two scattered wave directions; Γ is the coupling coefficient

$$\Gamma=\Gamma_H+\Gamma_{EM} \tag{3}$$

Γ_H and Γ_{EM} denote the hydrodynamic coupling component and electromagnetic coupling component.

$$\Gamma_H=-\frac{i}{2}\left[k+k'-\frac{(kk'-\vec{k}\vec{k'})(\omega^2+\omega_B{}^2)}{mm'\sqrt{kk'}(\omega^2-\omega_B{}^2)}\right] \tag{4}$$

$$\Gamma_{EM}=\frac{1}{2}\left[\frac{(\vec{k}\vec{k_0})(\vec{k'}\vec{k_0})/k_0{}^2-2\vec{k}\vec{k'}}{\sqrt{\vec{k}\vec{k'}}-k_0\Delta}\right] \tag{5}$$

Where $\Delta=0.011-0.012i$ is the sea surface normalized impedance.

2.2 Linearization of the Second-order Radar Cross-section Equation

Due to the second-order scattering between the radar beam and the ocean waves, two columns of wave vectors are generated, resulting in a non-linear second-order radar cross-section equation. Directly involving this equation in inversion would increase computational complexity, thus simplification is required. The direction of one set of waves in the second-order interaction generated by the radar wave vectors is approximately equal to the direction of the Bragg wave vector[7,8]. So the equation can be linearized as follows:

$$\sigma^{(2)}{}_{m,m'}(\omega)=2^{8}\pi k_0^4|\Gamma|^2\int_0^{2\pi}S(m\vec{k})S(-2m'\vec{k_0})\frac{(2k_0)^4}{k'*^4}y*^3\left|\frac{\partial y}{\partial h}\right|_{\theta,y=y*}d\theta \tag{6}$$

where $k'*=(y*^4+4y*^2k_0\cos\theta+(2k_0)^2)^{1/2}$.

To eliminate the influence of path loss on the inversion results, the first peak on the stronger side of the Doppler spectrum and the second-order spectrum region are selected[9,10]. The second-order spectrum is then divided by the first peak to obtain a normalized second-order spectrum.

$$R_{m'm}(\omega)=\frac{\sigma^{(2)}(\omega)}{\int\sigma^{(1)}(\omega)d\omega}=\int_0^{2\pi}C(\theta)S(m\vec{k})d\theta \tag{7}$$

where $C(\theta)=\left|\Gamma\right|^2(4(2k_0)^4y^{*3})/((y^{*4}+4y^{*2}k_0\cos\theta+(2k_0)^2)^2)\left|(\partial y)/(\partial h)\right|_{\theta,y=y^*}$.

3. Inversion Method For Wave Spectrum with Wind And Swell Coexisting

The steps of the algorithm for multi-frequency HF radar inversion of mixed waves (wind waves and swell waves) are as follows:

1) Extract the first-order peak and outer second-order spectrum of the stronger side of the Doppler spectrum of the mixed waves.
2) To calculate the ratio of the energy between the extracted outer second-order spectrum and the first-order peak, denoted as , where the subscript represents its corresponding frequency.
3) Multiply the obtained energy ratio by the directional distribution factor and integrate it from to to obtain the matrix of the omnidirectional wave spectrum coefficients

$$C_f=\int_0^{2\pi}C_f(\theta)G(\theta)d\theta \tag{8}$$

where $G(\theta)$ is the orientation distribution function.

4) Repeat steps 2) and 3) for each radar frequency and form the matrix by combining the coefficient matrices of different radar frequencies.

$$R=[R_{f_1}(\omega)\ \ R_{f_2}(\omega)\ R_{f_3}(\omega)\ R_{f_4}(\omega)]^T \tag{9}$$

$$C=[C_{f_1}\ \ C_{f_2}\ C_{f_3}\ C_{f_4}]^T \tag{10}$$

5) Extract the main features of the coefficient matrix using Principal Component Analysis (PCA) and calculate its pseudoinverse to obtain the omnidirectional wave spectrum.

$$S(k)=C^{+}R \tag{11}$$

where C^{+} is the pseudo-inverse of the coefficient matrixC.

4. Simulation Analysis

The target spectrum of a mixed wave consists of the wind wave spectrum and the surge spectrum, where the PM spectrum is selected as the target spectrum of the wind wave with the fo1lowing equation.

$$f(k)=\frac{0.005e^{-0.74(k_c/k)^2}}{k^4} \tag{12}$$

Directional Distribution Factor Selection Cardioid Directional Distribution Factor

$$G(\theta)=\cos^4(\frac{\theta-\theta^*}{2})\Big/\int_{-\pi}^{\pi}\cos^{2s}(\frac{\theta}{2})d\theta \tag{13}$$

The target spectrum of the swell is

$$S_s(k,\theta)=Hs^2k^{-1}\delta(k-k_s)\delta(\theta-\theta_s) \tag{14}$$

The effective wave spectrum for a mixed wave can be obtained from the above

$$S(k,\theta)=f(k)G(\theta)+S_s(k,\theta) \tag{15}$$

Fig. 1 shows the Doppler spectra obtained from the mixed waves at 13 m/s for different radar frequencies: 8, 13, 19, and 25MHz. The first-order peaks and second-order spectral regions of the wind waves can be seen from the simulated Doppler spectra, and also the swell information is obvious. Fig 2(a) shows the inversed wave spectrum and Fig 2(b) represents the corresponding residual spectrum. The blue line and the yellow-dotted line represent the inversed wave spectra by the new method and the method described in [11], respectively. And the orange-dashed line is the target spectrum. It can be seen that both the wind and swell information can be extracted by the new method.

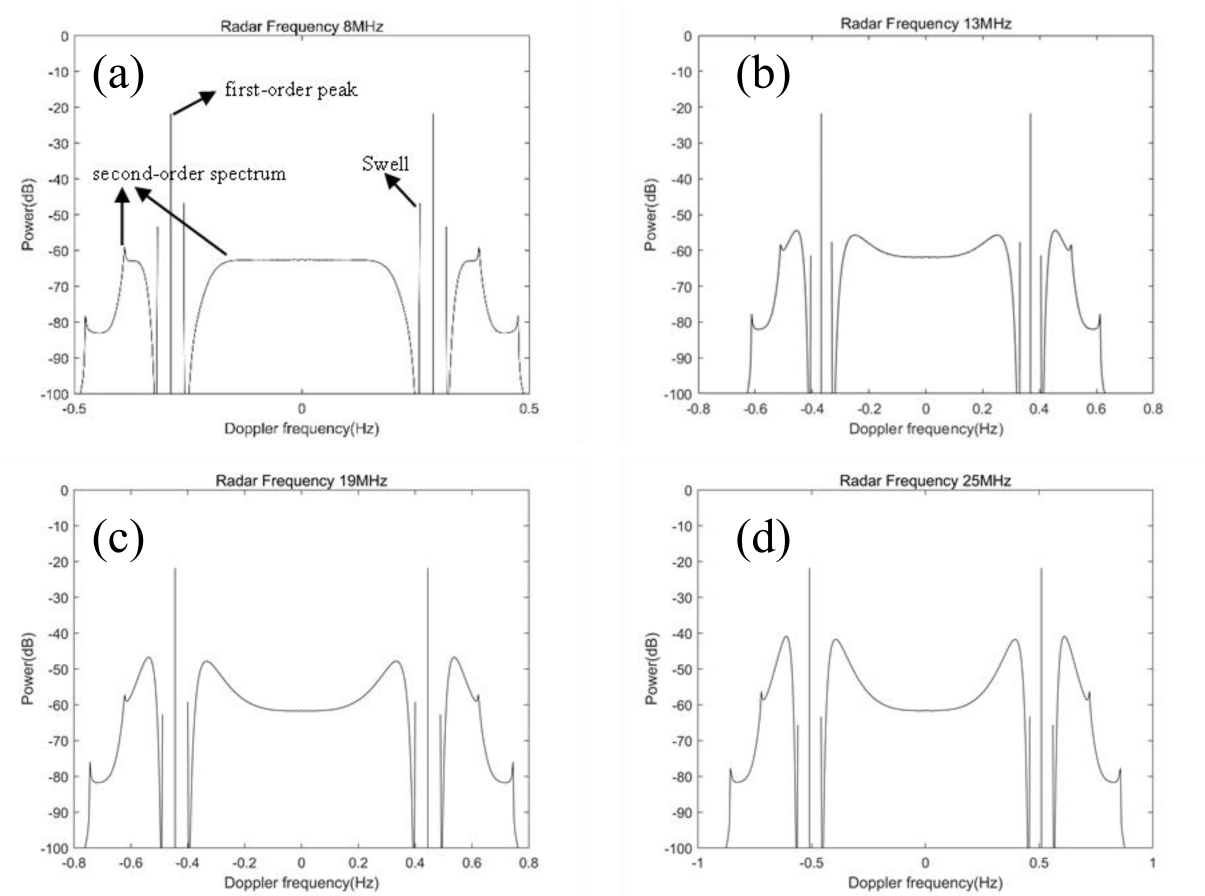

Fig. 1 Doppler spectra obtained from the mixed waves at 13 m/s for different radar frequencies. (a) 8MHz; (b) 13 MHz (c) 19 MHz; (d) 25MHz.

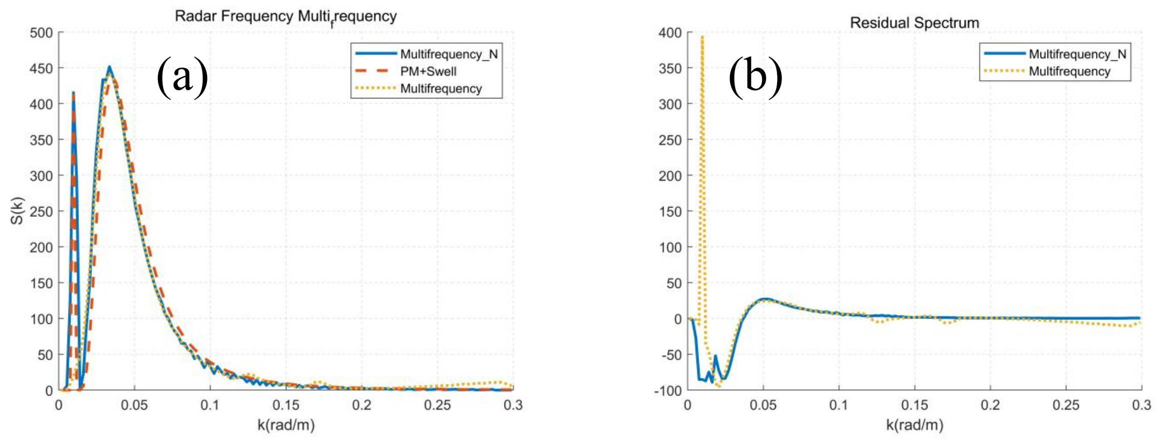

Fig. 2 Comparison of the inverted nondirectional wave spectrum and the theoretical spectrum when the wind speed is 13 m/s. (a) Nondirectional wave spectra. PM+Swell denotes the theoretical target spectrum. Multifrequency_N and Multifrequency represent the inversed wave spectra by the new method and the method described in [11], respectively. (b) Residual spectra.

A simulation for verification of the new method at a wind speed of 9m/s is also given. Fig. 3 shows the Doppler spectra and Fig. 4 shows the corresponding inversed wave spectra . Similarly, the new method yielded better results.

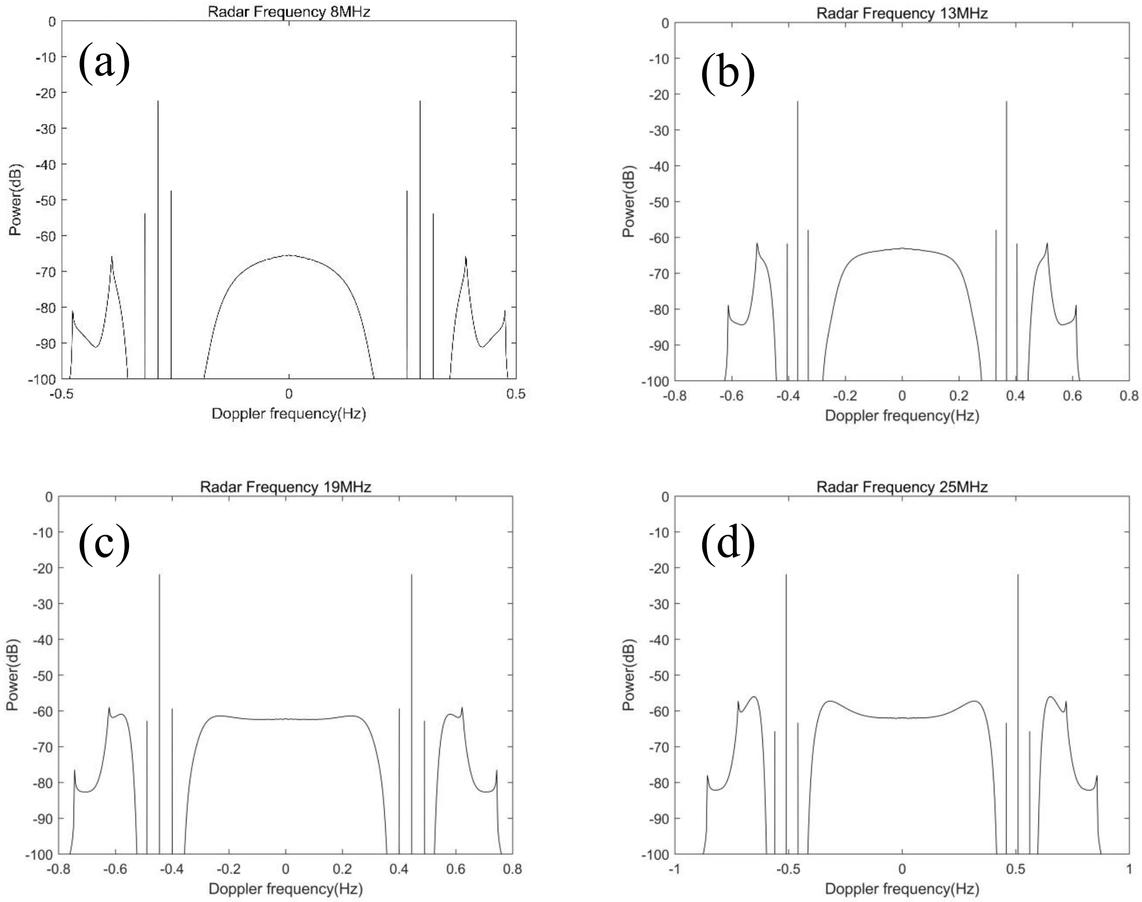

Fig. 3 Doppler spectra obtained from the mixed waves at 9 m/s for different radar frequencies. (a) 8MHz; (b) 13 MHz (c) 19 MHz; (d) 25MHz.

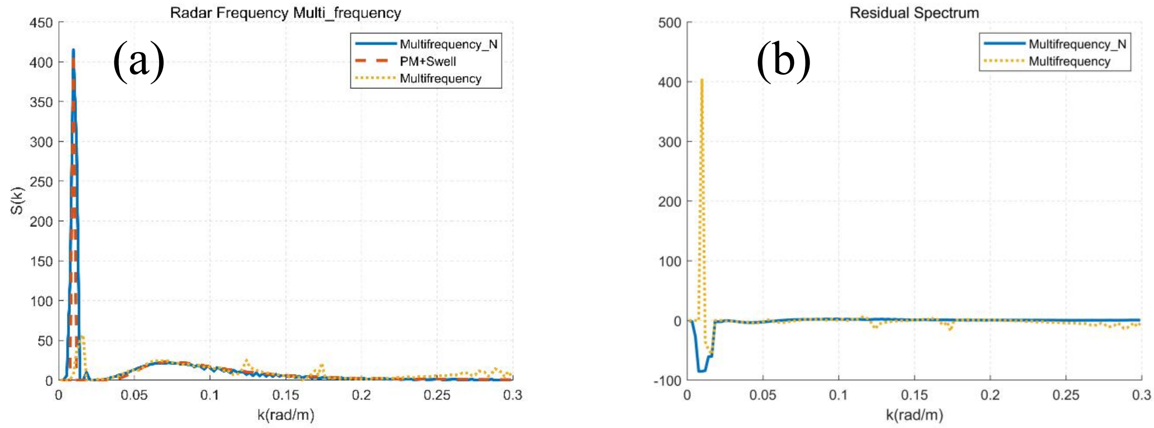

Fig.4 Comparison of the inverted nondirectional wave spectrum and the theoretical spectrum when the wind speed is 9 m/s. (a) Nondirectional wave spectra. (b) Residual spectra.

5. Summary

In this paper, under the complex sea conditions with wind and swell coexisting, a method for the ocean wave spectrum extraction has been formulated by utilizing multi-frequency HF radar sea echoes. Moreover, the proposed method is verified by the simulated multi-frequency HF radar sea echoes. The simulation results show that the proposed method can exactly inverse the ocean wave spectrum with wind and swell coexisting.

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