Research on Underwater Straightness Measurement Method Using Ropes

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Abstract. In this paper, we study the behavior of a rope impacted by currents in underwater crossflow and propose a method for measuring the straightness of long distance underwater objects using the rope as a reference line. The theoretical model includes the behavior of ropes under different loads and the composition of loads, which can simulate the displacement state of ropes in actual sea areas and be used for straightness measurement. This method provides an idea for conducting long-distance straightness measurement underwater to meet practical needs.

Keywords: Straightness measurement, Drag force, Rope behavior, Theoretical model.

1. Introduction

The measurement of Straightness error has been a consistent topic in the field of geometric measurement for a long time and is considered as one of the most important and fundamental aspects. The conventional approach to measure straightness involves comparing it with an ideal element, such as a ruler, tensioned steel wire, or physical forms like laser collimators based on optical principles or levels made using horizontal physical principles. Besides, parameters measuring principles and effective boundary control principles can also be employed for this purpose[1,2], but their usage is not popular in practical scenarios.

There are numerous methods available to measure straightness on land, but when it comes to underwater measurements, various problems arise, such as obstructed vision, arduous operation, and water flow disturbances. Typically, a laser is used on land to measure the straightness of long-distance object arrangements. Unfortunately, laser beams attenuate rapidly underwater. Even blue-green wavelengths, which have the least absorption, can only spread for tens of meters [3]. Therefore, using laser beams as reference lines is not feasible.

This paper aims to examine the force and displacement of ropes made from various materials under different water flow conditions. It proposes a novel method of measuring straightness by using the rope as a reference line, and establishes a simple model based on theoretical derivation. This model can accurately predict the shape of the rope when it is impacted by water flow, thus providing information about any deviation from the ideal straight line.

2. Theoretical Model

This part is the theoretical model derivation of the force behavior of ropes, and all these calculations have been made under ideal conditions with no allowance for deformation in the rope itself. However, in practical applications, such as underwater straightness measurement targeted by this paper, the rope deforms as it faces gravity, buoyancy, and the impact force of water flow.

Therefore, to account for these aspects, tension will be applied at both ends to produce stress stiffening effect in the rope.

When a rope is submerged in water, it undergoes axial tension and the load is divided into two components - radial and axial. The shape of the rope under tension is usually linear, albeit slightly curved. Let us consider the connecting line at both ends of the rope as the X-direction, and the Y-direction perpendicular to the X-direction. To begin with, we will derive it on the X-Y plane. Additionally, we will take a tiny portion of the rope, i.e., d*x*, for stress analysis. The angle between the rope and the x-axis is denoted by θ, while the length of the rope is d*l*.

2.1 Radial uniform load only

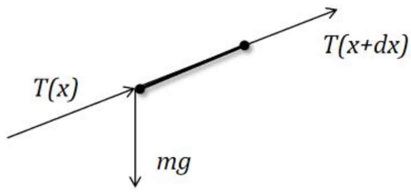

Fig 1. The rope with gravity only

Assuming that the rope will not deform (due to an infinite elastic modulus) and is characterized by a soft, uniform thickness and mass distribution, let's examine the simplest scenario. In this case, the rope is placed under fixed tension t at both ends, with only gravity acting as a load in the vertical direction besides the axial tension. To begin our analysis, we take a small section dx from the rope and consider its length d*l*.

$$dl = \frac{dx}{\cos\theta} = \sqrt{1+\tan^2\theta}dx = \sqrt{1+y'^2}dx\#(1)$$

The tension at both ends will be transmitted through the rope, and this section will be pulled by the previous section of rope as

$$T(x+dx) = \left(T_x(x+dx), T_y(x+dx)\right)\#(2)$$

where $T_y = T_x y'$.The gravity received is

$$mg = \frac{\rho\pi D^2}{4} gdl = \frac{\rho\pi D^2}{4} g\sqrt{1+y'^2}dx\#(3)$$

Where ρ Is the density, D is the diameter of the rope, and the pull of this section on the rear rope is

$$T(x) = \left(T_x(x), T_y(x)\right)\#(4)$$

In the equilibrium state, the resultant force on this section of rope is

$$F = T(x+dx) - T(x) + mg = \left(\frac{dT_x}{dx}dx, \frac{dT_y}{dx}dx - \frac{\rho\pi D^2}{4}g\sqrt{1+y'^2}dx\right) = (0,0)\#(5)$$

In the X direction, there is $\frac{dT_x}{dx} = 0$, and $T_x = c$ can be obtained, that is, the axial tension on this section of rope is a fixed constant. In the Y direction, there is $\frac{dT_y}{dx} = \frac{\rho\pi D^2}{4}g\sqrt{1+y'^2}$, and because $T_y = T_x y' = cy'$, there is

$$\frac{dT_y}{dx}=\frac{d(T_x y')}{dx}=\frac{cdy'}{dx}=cy''=\frac{\rho\pi D^2}{4}g\sqrt{1+y'^2}\#(6)$$

Collate the above formula to get

$$y''=\frac{\rho\pi D^2 g}{4c}\sqrt{1+y'^2}\#(7)$$

By solving the differential equation, we can obtain

$$y=\frac{4c}{\rho\pi D^2 g}\cosh\left(\frac{\rho\pi D^2 g}{4c}x+c_1\right)+c_2\,\#(8)$$

where *c* means the tension value *T* at both ends, c_1 and c_2 can be determined by the two points fixed by the rope, for example, the coordinates (0, 0) and (*l*, 0) can be taken. At this time, the displacement function is

$$y=\frac{4T}{\rho\pi D^2 g}\cosh\left(\frac{\rho\pi D^2 g}{4T}x-\frac{\rho\pi D^2 g}{8T}l\right)-\frac{4T}{\rho\pi D^2 g}\cosh\left(\frac{\rho\pi D^2 g}{8T}l\right)\#(9)$$

where *T* is the tension at both ends, *ρ* Is the rope density, *D* is the rope diameter, *g* is the gravitational acceleration, and *l* is the distance between the fixed points at both ends of the rope.

2.2 Uniform load in any direction

In this case, it is assumed that the load on the rope is uniform, but there is an axial (X-direction) component, and the uniform load on *dx* is

$$T_c=(T_{cx},T_{cy})=(adx,bdx)\#(10)$$

The tension at both ends will be transmitted through the rope, and this section will be pulled by the previous section of rope as

$$T(x+dx)=\left(T_x(x+dx),T_y(x+dx)\right)\#(11)$$

where $T_y=T_x y'$.The tension of this section on the subsequent rope is

$$T(x)=\left(T_x(x),T_y(x)\right)\#(12)$$

so the resultant force on d*x* segment is

$$F=\left(\frac{dT_x}{dx}dx-T_{cx},\frac{dT_y}{dx}dx-T_{cy}\right)=\left(\frac{dT_x}{dx}dx-adx,\frac{dT_y}{dx}dx-bdx\right)=(0,0)\#(13)$$

There is $\frac{dT_x}{dx}dx=adx$ in the X direction, so $T_x=ax+c_0$. And there is $\frac{dT_y}{dx}dx=bdx$ in the Y direction, and because $T_y=T_x y'=(ax+c_0)$, there is

$$\frac{(ax+c_0)dy'+y'd(ax+c_0)}{dx}=b\#(14)$$

Collate the above formula to get

$$y''+\frac{a}{ax+c_0}y'=\frac{b}{ax+c_0}\#(15)$$

By solving the differential equation, we can obtain

$$y=\frac{b}{a}x+\frac{c_1-\frac{b}{a}c_0}{a}\ln\left(ax+c_0\right)+c_2\#(16)$$

where a and b are determined by the load, c_0 is the given tension T, c_1 and c_2 can be determined by the two points fixed by the rope. If the coordinates of the two fixed points are (0, 0) and (l, 0), the displacement function is

$$y=\frac{b}{a}x+\frac{-\frac{b}{a}l}{\ln\left(\frac{al+T}{T}\right)}\ln\left(ax+T\right)+\frac{\frac{b}{a}l}{\ln\left(\frac{al+T}{T}\right)}\ln T\#(17)$$

2.3 Uneven load in any direction

In this case, the load on the rope is uneven, which will change in size or direction with the change of x. Assume that the distribution of load on dx section is

$$T_c=\left(\frac{T_{cx}(x)}{dx}dx,\frac{T_{cy}(x)}{dx}dx\right)\#(18)$$

The tension at both ends will be transmitted through the rope, and this section will be pulled by the previous section of rope as

$$T(x+dx)=\left(T_x(x+dx),T_y(x+dx)\right)\#(19)$$

where $T_y=T_xy^{'}$ .The tension of this section on the subsequent rope is

$$T(x)=\left(T_x(x),T_y(x)\right)\#(20)$$

So the resultant force on dx segment is

$$F=\left(\frac{dT_x}{dx}dx-\frac{dT_{cx}(x)}{dx}dx,\frac{dT_y}{dx}dx-\frac{dT_{cy}(x)}{dx}dx\right)\#(21)$$

There is $T_x=T_{cx}(x)+c_0$ in the X direction, $\frac{dT_y}{dx}dx=dT_{cy}(x)$ in the Y direction, and because $T_y=T_xy^{'}=(T_{cx}(x)+c_0)y^{'}$, there is

$$\frac{(T_{cx}(x)+c_0)dy^{'}+y^{'}d(T_{cx}(x)+c_0)}{dx}=\frac{dT_{cy}(x)}{dx}\#(22)$$

Collate the above formula to get

$$y^{''}+\frac{T_{cx}^{'}(x)}{T_{cx}(x)+c_0}y^{'}=\frac{T_{cy}^{'}(x)}{T_{cx}(x)+c_0}\#(23)$$

By solving the differential equation, we can obtain

$$y=\int\frac{T_{cy}(x)+c_1}{T_{cx}(x)+c_0}dx+c_2\#(24)$$

where c_0 is the given tension T, c_1 and c_2 can be determined by the two points fixed by the rope.

2.4 Rope behavior in three-dimensional coordinates

Considering the three-dimensional coordinates, let the distribution of load on *dx* section be

$$T_c = \left(\frac{T_{cx}(x)}{dx}dx,\ \ \frac{T_{cy}(x)}{dx}dx,\ \ \frac{T_{cz}(x)}{dx}dx\right)\#(25)$$

The tension at both ends will be transmitted through the rope, and this section will be pulled by the previous section of rope as

$$T(x+dx) = \left(T_x(x+dx),\ \ T_y(x+dx),\ \ T_z(x+dx)\right)\#(26)$$

Similar to the previous method, we can obtain

$$y = \int \frac{T_{cy}(x)+c_1}{T_{cx}(x)+c_0}dx + c_2\,\#(27)$$

$$z = \int \frac{T_{cz}(x)+c_3}{T_{cx}(x)+c_0}dx + c_4\,\#(28)$$

where c_0 is the given tension T, and c_1, c_2, c_3 and c_4 can be determined by the two points fixed by the rope.

The total rope offset is

$$h = [(\int \frac{T_{cy}(x)+c_1}{T_{cx}(x)+c_0}dx + c_2)^2 + (\int \frac{T_{cz}(x)+c_3}{T_{cx}(x)+c_0}dx + c_4)^2\,]^{\frac{1}{2}}\#(29)$$

2.5 Composition of the load

Investigated the load composition of *dx* section, the force on the rope in the water is mainly composed of gravity, buoyancy and drag force generated by water flow.

In the Z direction, the rope is subjected to gravity and buoyancy in the water

Gravity is

$$mg = \frac{\rho_1 g\pi D^2}{4}dx\#(30)$$

where ρ_1 is the density of the rope.

Buoyancy is

$$F_f = \frac{\rho_0 g\pi D^2}{4}dx\#(31)$$

where ρ_0 is the density of seawater.

Gravity and buoyancy only appear in the vertical direction, and the resultant force in the Z direction is

$$Fz = \frac{(\rho_1-\rho_0)g\pi D^2}{4}dx\#(32)$$

Therefore, if the density of the rope is close to that of the sea water, the resultant force in the Z direction can be reduced.

In addition, it will be subject to the force from the water flow

$$F_w = \frac{C_D\rho_0 U^2}{2}D\cos\theta\, dx\#(33)$$

where C_D is the drag force coefficient, U is the water flow velocity, θ Is the included angle between the water flow and the radial direction of the rope. When the water flow is perpendicular to the rope, it is 0, and when it is along the rope direction, it is 90° at most.

If only the drag force generated by the water flow perpendicular to the rope is considered and gravity is not considered, F_w is used to replace mg and θ Is 0, we can regard F_w as a uniform force field, then the displacement function becomes

$$y = \frac{2T}{C_D\rho_0 U^2 D}\cosh\left(\frac{C_D\rho_0 U^2 D}{2T}x - \frac{C_D\rho_0 U^2 D}{4T}l\right) - \frac{2T}{C_D\rho_0 U^2 D}\cosh\left(\frac{C_D\rho_0 U^2 D}{4T}l\right)\#(34)$$

2.6 Change in rope shape

Considering the deformation of the fiber rope, the deformation of the rope is mainly caused by the axial tension when a huge tension force is applied at both ends. The tensile deformation is

$$\Delta l = \frac{Tl}{EA}\#(35)$$

where T is the axial tension, E is the elastic modulus, and A is the sectional area.

Since the drag force of the water flow is very small compared with the tension force at both ends, the radial strain of the rope is mainly caused by the axial strain.

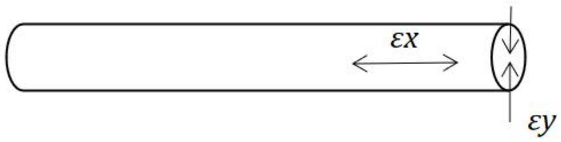

Fig 2. Axial and radial deformation

The axial strain is

$$\varepsilon x = \frac{\Delta l}{l} = \frac{T}{EA}\#(36)$$

Assuming that the deformation is approximately uniform, the radial strain is

$$\varepsilon y = -\mu \cdot \varepsilon x = -\mu\frac{T}{EA}\#(37)$$

where μ is Poisson's ratio. The diameter after deformation is

$$D^{'} = \left(1 - \frac{\mu T}{EA}\right)D = \left(1 - \frac{4\mu T}{\pi E D^2}\right)D = D - \frac{4\mu T}{\pi E D}\#(38)$$

The deformation will change the diameter, and then affect the linear density and the force affected by the water flow, so as to affect the offset. At this time, the gravity becomes

$$mg = \frac{\rho_1 g\pi}{4}\left(D - \frac{4\mu T}{\pi E D}\right)^2 dx\#(39)$$

Buoyancy becomes

$$F_f = \frac{\rho_0 g\pi}{4}\left(D - \frac{4\mu T}{\pi E D}\right)^2 dx\#(40)$$

The flow force becomes

$$F_w = \frac{C_D\rho_0 U^2}{2}\left(D - \frac{4\mu T}{\pi E D}\right)\cos\theta\, dx\#(41)$$

3. Summary

This paper presents a study on the displacement behavior of cables in crossflow, and proposes a new theoretical method for measuring straightness based on cable displacement. Once the displacement state of the rope is known, the influence caused by water impact can be calculated and compensated, with the accuracy of calculation determining the accuracy of straightness measurement. Subsequent simulation calculations and flume experiments have preliminarily verified its accuracy.

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