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The number of 8n+1 primes compare with 8n-1 primes

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Abstract. There is a phenomenon in mathematics: there is a phenomenon that before a natural number K, primes of the form 4n+1 do not appear more frequently than 4n-1 primes; Beyond k, between k and k+k', the above phenomenon is reversed, the frequency of 4n+1 primes is not less than 4n-1 primes; After exceeding k+k', between k+k 'and k+k'+k", it is reversed again ...... The J.E. Littlewood proved the first stage of the phenomenon: primes of the form 4n+1 appear no more frequently than 4n-1 primes before a natural number k. In this paper used a more easy method and directly prove the phenomenon very shortly , provides a theoretical proof for this description.This method is more easy directly and elementary than Littlewood' ,and It can help people understand this phenomenon better, and at the same time, it provides a good example for the optimization of number theory research methods and the use of some elementary methods to study mathematical problems. At the same time, there is a generalization conjecture: before a natural number K, which of 8n+1 and 8N-1 primes appear more frequently? The conjecture remains unsolved. Littlewood proved the occurrence frequency theorem of 4n+1 primes and 4N-1 primes, and this paper also gave the proof, the method is different from Littlewood, but he was the first; However, for 8n+1 primes compare with 8n-1 primes, we prove for the first time that the result is same as 4n+1 primes compare with 4n-1 primes.

Keywords: Number theory, P =4n±1, P = 8n±1.

1. Introduction

American Mathematics Monthly[1] published an article titled "Prime Number Races"[2], which introduced a mathematical conjecture: The numbers of primes of the form of 4n+1 and 4n-1 which one is more?

We use a table in the article "Prime Number Races" :

Table 1. The number of the form 4n + 1 primes and 4n -1 primes up to x .

x	Number of primes 4n -1 up to x	Number of primes 4n + 1 up to x
100	13	11
200	24	21
300	32	29
400	40	37
500	50	44
600	57	51
700	65	59
800	71	67
900	79	74
1000	87	80
2000	155	147
3000	218	211
4000	280	269
5000	339	329
6000	399	383
7000	457	442
8000	507	499
9000	562	554
10,000	619	609
20,000	1136	1125
50,000	2583	2549
100,000	4808	4783

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Theorem (J.E. Littlewood[3], 1914). There are arbitrarily large values of x for which there are more 4n + 1 primes up to x than 4n - 1 primes.

This is clearly a very low percentage, but it is not evident from the limited data available

Tends to zero. A few years ago Richard Guy published a wonderful article in the MONTHLY entitled "The Law of Small Numbers" [4]. In the article, Guy points out several fascinating phenomena that are "obvious" for small integers but disappear when studying large integers.

At the same time, there is a generalization conjecture: before a natural number K, which of 8n+1 and 8n-1 primes appear more frequently? We proved for the first time that the result is same as 4n+1 primes compare with 4n-1 prime.

2. reasoning process

Lemma 1: If 2N≠2mn+m+n, then 2N+1 is a prime number[5]

Prove: If N=2mn+m+n, then 2N+1=2 (2mn+m+n)+1=4mn+2m+2n+1=(2m+1)(2n+1), so 2N+1 is a composite number. If N≠2mn+m+n, then 2N+1≠(2m+1)(2n+1), 2N+1 is a prime number.

Lemma 2: If N≠8ab+6a+2b+1, 2N+1 is a prime number

Prove: If 2mn+m+n is odd number,

Make m=2a, n=2b+1,2mn+m+n=8ab+6a+2b+1, so if N=8ab+6a+2b+1,

N=2mn+m+n, according to Lemma 1, get 2N+1 is a composite, if N≠2(4ab+3a+b)+1, then 2N+1 is a prime .

Lemma 3: If N=8ab+2a+2b, 2N+1 is a composite; If N=8ab+6a+6b+4,so 2N+1 is a composite.

Prove: For 2mn+m+n is even number, let m=2a, n=2b, 2mn+m+n=2(2a)(2b)+2a+2b=8ab+2a+2b, so if N=8ab+2a+2b, N=2mn+m+n, according to Lemma 1, so 2N+1 is a composite, if N≠8ab+2a+2b, 2N+1is a prime number.

For 2mn+m+n, let m=2a+1, n=2b+1, 2mn+m+n=8ab+6a+6b+4, so if N=8ab+6a+6b+4, N=2mn+m+n, 2N+1is a composite number, if N≠8ab+6a+6b+4, 2N+1is a prime number.

Theorem: For 8n+1 primes compare with 8n-1 primes, the result is same as 4n+1 primes compare with 4n-1 primes.

Prove:4n-1=2(2n-1)+1, 2n-1 is odd numb; 4n+1=2(2n)+1, 2n-1 is even number. According to Lemma 2, 2mn+m+n is odd number, let m=2a, n = 2b+1, then 2mn +m+n = 8 ab+6a +2b+1; According to Lemma 3, 2mn+m+n is even number, let m=2a, n=2b, then 2mn+m+n=8ab+2a+2b, let m=2a+1, n=2b+1, then 2mn+m+n= 8 ab+6 a+6 b+4.

Use N1, N2 and N3 to represent the set of positive integers satisfying the following conditions respectively.

8ab+2a+2b	N1
8ab+6a+6b+4	N2
8ab+6a+2b+1	N3

There are more composite numbers of the form as 2(2n)+1 than the form of 2(2n-1)+1 (N1+N2 > N3), that is, there are more 4n+1 composite numbers than 4n-1 composite numbers, so before a number K, the number of 4n+1 primes is less than 4n-1 primes.

Note: For conclusion 1, it should be noted that N1, N2 and N3 have their own repeats, For 8ab+2a+2b, 8ab+6a+6b+6 and 8ab+6a+2b+1, As the values become larger and larger, their own repeats become more and more. Therefore, before the repetition degree does not exceed the difference in the number of sets, that is, before a natural number k, 4n+1 primes occur no more frequently than 4n-1 primes. The the first stage of the conjecture introduced in the introduction is proved.

Also there is an unsolved conjecture： 8n+1 of primes and 8n-1 of primes which one is more,before a large number k？

8n+1=2 (4n) +1, 8n-1=2 (4n) -1, for 2N-1 is a prime , then N must meet N=2mn+m+n+1, is also：N=

8ab+2a+2b+1

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8ab+6a+6b+5

8ab+6a+2b+2

For 4n is even number,so 8n+1=2 (4n) +1,4n must meet 8ab+2a+2b and 8ab+6a+6b+4, 8n-1=2 (4n) -1,this 4n must meet 8ab+6a+2b+2,due to 8ab+2a+2b < 8ab+6a+2b+2,so the 4n that can be expressed as 8ab+2a+2b and 8ab+6a+6b+4 are more than 8ab+6a+2b+2.Even numbers of 4n or not 4n of 8ab+2a+2b is equall ,so for 8ab+6a+2b+2 is also equall (reason:8ab+6a+2b+2=8ab+2a+2b+4a+2).

Note: For conclusion 2, it should be noted that 8ab+2a+2b 、8ab+6a+6b+4 and 8ab+6a+2b+2 also have their own repeats, As the values become larger and larger, their own repeats become more and more. Therefore, before the repetition degree does not exceed the difference in the number of sets, that is, before a natural number k, 8n+1 primes occur no more frequently than 8n-1 primes.

3. Conclusion

For 8n+1 primes compare with 8n-1 primes, the result is same as 4n+1 primes compare with 4n-1 primes.

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