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Study on Calculation of Minimum Thickness of Sample in Thermal Conductivity Measurement

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Abstract. It is found that the true thermal conductivity can be measured only when material reaches a certain minimum thickness, within which the thermal conductivity varies with thickness. Based on the derivative of Fourier's law, it is pointed out that the phenomenon of thermal conductivity varying with thickness is caused by the existence of second derivative of temperature. Both of the FTC (function of thermal conductivity) and the calculation formula of MMT (minimum measured thickness) are established. For materials of GFRP (glass fiber reinforced plastic), neoprene and silicone, their R-squared of FTC are respectively 0.9995, 0.9668 and 0.9976, and their MMTs are 5.2287 mm, 4.7034 mm and 12.7532 mm, respectively. What's more, their interfacial thermal conductivities are different, which reveals that thermal conductivity of material is a numerical field.

Keywords: measurement of thermal conductivity; Fourier' s law; minimum measurable thickness.

1. Introduction

Thermal conductivity is a significant parameter to characterize the thermal properties of materials, as well as a basis to measure whether materials can adapt to specific thermal processes. Moreover, it is the basic data for analysing and calculating specific thermal processes and engaging in engineering design. Thermal conductivity plays a key role in thermal design [1, 2], thermal insulation structure design [3], temperature prediction [4] and other applications [5]. Therefore, it is very important to accurately measure the thermal conductivity of materials.

Thermal conductivity measurement methods [6] include steady-state and transient techniques. The steady-state techniques include hot flat plate, radial heat flow, comparative cut bar and heat flow meter, etc. The transient techniques include hot-strip, hot-wire, thermal probe and laser flash etc. In recent years, it has been noted, in the thermal conductivity measurements of transient methods [7, 8, 9], that measurement results of thermal conductivity change with the thickness if the sample thickness is less than minimum measured thickness (MMT). The measured thermal conductivity is true only when the sample reaches and exceeds MMT.

The phenomenon of thermal conductivity varying with thickness has attracted a lot of attention from many researchers for a long time. A number of models, including molecular dynamics (MD) and wave vector (WV), have been established to calculate the thermal conductivity of materials. Alder & Wainwright proposed (1957) and developed (1957) MD model [10]. Depending on whether the simulation cell requires a temperature gradient, molecular dynamics is divided into equilibrium molecular dynamics (EMD) and non-equilibrium molecular dynamics (NEMD). EMD model does not require a temperature gradient, and Green-Kubo relationship is used as calculation formula of thermal conductivity [11]. NEMD model needs to establish a temperature gradient in the object and then directly calculate the thermal conductivity according to Fourier law, which is similar to experimental measurement method [12]. In 2004, Chantrenne & Barrat proposed WV model to analyse the thermal conductivity of nanostructures [13]. WV model considers all phonon properties: modulus, group velocity, relaxation times due to the Umklapp process and scattering at boundary surfaces, and their variation with the direction of the wave vector. In 2007, Wang et al. introduced the relaxation time of the Normal process and the second-order-3 photon process to modify WV model [14].

Although MD and WV model can predict the change of the thermal conductivity of object at Micro/Nano scale, it is still in lack of relevant theoretical explanations for thermal conductivity

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varying with thickness above millimetre scale. At present, the MMT of the sample can only be determined by experiment, and clear theoretical calculation formula of MMT cannot be given.

Based on Fourier's law, this paper aims to give the calculation formula of MMT, and establish mathematical model of thermal conductivity varying with thickness, which can be applied not only to the Micro/Nano scale, but also to the millimetre scale.

2. Theory and methods

2.1 Theoretical analysis

Thermal conductivity is a thermophysical parameter defined by Fourier's law, so the investigation of thermal conductivity is inseparable from Fourier's law.

$$q=-\lambda\frac{dT}{dy} \tag{1}$$

where q= heat flux, W/m2; λ= thermal conductivity, W/(m·K); and dT/dy = temperature gradient, K/m.

To study the rule of thermal conductivity varying with thickness, it is necessary to take the derivative of Fourier law.

$$\frac{dq}{dy}=-\left(\frac{dT}{dy}\cdot\frac{d\lambda}{dy}+\lambda\cdot\frac{d^2T}{dy^2}\right) \tag{2}$$

At constant heat flux, we have dq/dy=0. By dividing Equation 2 by Equation 1, heat flux, thermal conductivity and temperature gradient can be separated.

$$\frac{dq}{qdy}=\frac{d\lambda}{\lambda dy}+\frac{d^2T}{dy^2}\Big/\frac{dT}{dy}=0 \tag{3}$$

Equation 3 can be expressed in two ways:

$$\frac{d\lambda}{\lambda dy}=-\frac{d^2T}{dy^2}\Big/\frac{dT}{dy}\begin{cases}\neq 0, & \dfrac{d^2T}{dy^2}\neq 0\\ =0, & \dfrac{d^2T}{dy^2}=0\end{cases} \tag{4}$$

Equation 4 reveals that the thermal conductivity varies with thickness due to the existence of the second derivative of temperature; When the second derivative of temperature does not exist, the thermal conductivity is a constant.

2.2 Discussion of data fitting methods

If we use polynomial series to fit the experimental data of these three materials. Although the fitting result is good, it is not in accordance with the definition of the thermal conductivity from Fourier’s law, since that the function of thermal conductivity (FTC) is always changing with the thickness of the sample. which So polynomial series fitting curves cannot be used to determine this minimum thickness. For this reason, polynomial series fitting curves cannot be used to determine the MMT.

As a matter of fact, thermal conductivity of the sample varies with thickness within the MMT. When the sample thickness is greater than the MMT, thermal conductivity does not change with thickness. In this regard, we should establish such a piecewise function: the thermal conductivity is a function of the thickness within the MMT; beyond the MMT, the thermal conductivity returns to a

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constant value, as shown in Figure 1. In this way, the MMT of the sample for thermal conductivity measurement can be accurately determined.

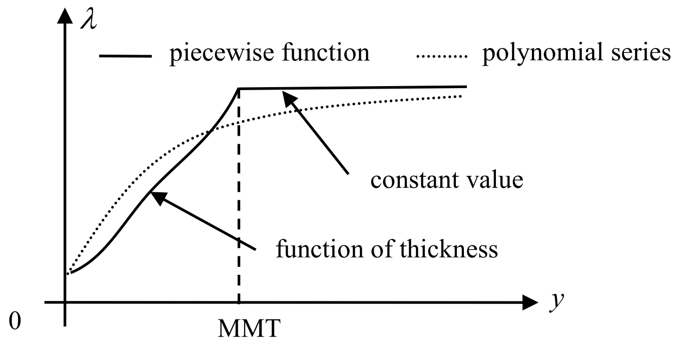

Figure 1. Thermal conductivity as a function of thickness.

In the experiment [7], hot-strip method was used to measure the thermal conductivity of glass fibre reinforced plastic (GFRP), neoprene and silicone at different thicknesses. The temperature difference required for measurement is provided by the hot strip in the form of interfacial heat flow. The total heat flow can be decomposed into sub-heat flows in three directions: length, width and thickness. The length and width of the sample are fixed and larger than the MMT, so its thermal conductivity remains unchanged in both the length and width directions. Therefore, we do not consider the partial heat flow in the length and width directions, but investigate the partial heat flow in the thickness direction.

The thermal conductivities of hot strip, GFRP, neoprene and silicone materials are respectively 16.75 W/(m·K), 0.45 W/(m·K), 0.0705 W/(m·K) and 1.37 W/(m·K). The experimental results are shown in Table 1, where the thermal conductivity corresponds to thickness of three materials in turn.

Table 1. Measurement results of thermal conductivity by the hot-strip method [7].

Materials	Thickness / mm	Thermal conductivity / $W \cdot m^{-1} \cdot K^{-1}$
GFRP	2.5/5/10/15/20	0.261/0.427/0.447/0.444/0.449
Neoprene	4/12/16/19/24	0.0665/0.0708/0.0708/0.0700/0.0704
Silicone	5/10/15/20	1.15/1.26/1.32/1.33

3. Solution

3.1 Function of thermal conductivity (FTC)

The key of this paper is to solve the rule that the thermal conductivity varies with the thickness within the MMT. It can be seen from Figure 1 that as the thickness of the material increases within the MMT, the value of the thermal conductivity returns from the small value to the original large value. We take

$$\frac{d\lambda}{\lambda dy} = A > 0 \tag{5}$$

In equation 5, since temperature and thermal conductivity, respectively, are independent physical quantity and physical property parameter, the negative number of the ratio of the second derivative of temperature to the first derivative is equal to the ratio of the first derivative of thermal conductivity to itself, so A is a constant value.

By separating variables and integrating Equation 5, we can get the FTC.

$$\lambda(y) = \lambda(0)\exp(Ay) \tag{6}$$

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where A= a new parameter, mm-1, only appearing in FTC, named as "recovery index" of thermal conductivity by us; λ(0)= interfacial thermal conductivity, W/(m·K).

Figure 2 shows the fitting results of the FTC of GFRP, neoprene and silicone. Their R-squared were 0.9995, 0.9668 and 0.9976, respectively. Different from polynomial series fitting, there are obvious intersection points between exponential curves and horizontal lines that do not vary with thickness, from which the minimum thickness can be accurately determined.

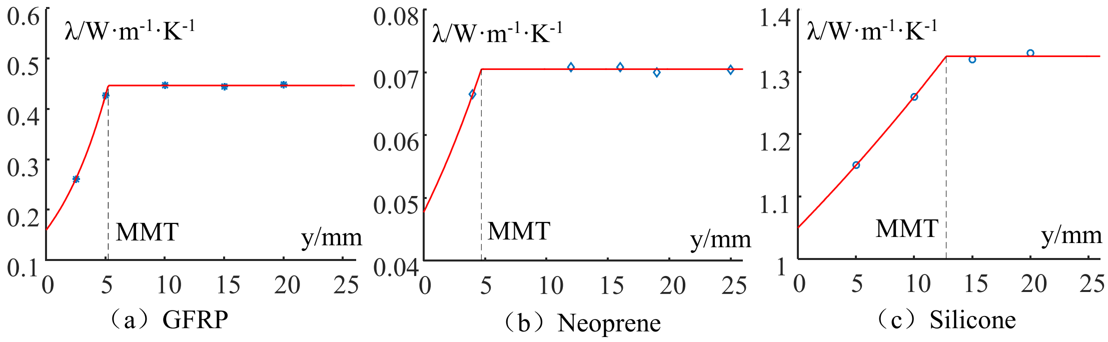

Figure 2. The curves fitted by the FTC of GFRP (a), neoprene (b) and silicone (c).

Table 2. Function fitting results for thermal conductivity experimental data

	GFRP	Neoprene	Silicone
λ(0) / W·m-1·K-1	0.1595	0.0477	1.0496
λ(MMT) / W·m-1·K-1	0.4467	0.0705	1.3250
MMT / mm	5.2287	4.7034	12.7532
A / mm-1	0.1969	0.0830	0.0183
δ / %	-0.30	0.00	-3.28
R2	0.9995	0.9668	0.9976

3.2 Minimum measured thickness (MMT)

The thermal conductivity of the sample increases exponentially with the thickness. At a certain position y= MMT, the thermal conductivity returns to the original large value, λ(MMT)= λmax. Thus, the MMT is obtained.

$$\mathrm{MMT} = \frac{1}{A}\ln\frac{\lambda_{\max}}{\lambda(0)} \tag{7}$$

Table 2 shows the MMT of GFRP, neoprene and silicone, which are 5.2287 mm, 4.7034 mm and 12.7532 mm, respectively. The relative error of thermal conductivity between fitted value and real value is small, which are -0.30%, 0 and -3.28%, respectively.

4. New cognition

4.1 Thermal conductivity

On the side of sample of different thickness, the thermal conductivity varies within a limited numerical range under the same heating power. On the side of hot strip, since heat conduction occurs at the interface of strip and sample. In order to ensure the existence of interfacial temperature gradient and continuity of the interfacial heat flow (Eq. 8), they can only share the same thermal conductivity at the interface. Therefore, the thermal conductivity of the three samples at the interface is equal to the hot strip.

$$\lambda\left(0^-\right)\cdot\left.\frac{dT}{dy}\right|_{0^-} = \lambda\left(0\right)\cdot\left.\frac{dT}{dy}\right|_{0} = \lambda\left(0^+\right)\cdot\left.\frac{dT}{dy}\right|_{0^+} \tag{8}$$

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$$\lambda_k(0) = \lambda_{hot(k)},\ k = 1,2,3 \tag{9}$$

where y= 0 is the interface between sample (superscript, "+") and strip (superscript, "-"). λhot(k) = thermal conductivity of hot strip, when interfacial heat conduction occurs between strip and material (k=1,2,3, respectively, representing GFRP, neoprene and silicone).

Table 2 shows the interfacial thermal conductivity of the three materials, which are 0.1595 W/(m·K), 0.0477 W/(m·K) and 1.0496 W/(m·K), respectively. This means that the thermal conductivity of hot strip varies at different heating powers, but λhot(k) ≤16.75 W/(m·K).

From what has been discussed above, the thermal conductivity of any object is a numerical field, and the thermal conductivity we know is usually the maximum.

4.2 Recovery index (A)

The FTC is expanded in series, as

$$\lambda(y) = \lambda(0)\exp(Ay) = \lambda(0)\cdot\left(1 + Ay + \frac{A^2}{2}y^2 + \cdots + \frac{A^n}{n!}y^n\right) \tag{10}$$

When A→0, we get Equation 11.

$$\lambda(y) = \lambda(0)\cdot(1 + Ay) \tag{11}$$

Apparently, the value of A is reflected in the exponential growth trend of FTC. When the value of A is small, the high-order term (An, n≥2) of the series can be ignored, and the exponential growth trend of the FTC is approximately linear. It is clear from Figure 2 that the value of A of FRP material is larger (0.1969 mm-1), and its thermal conductivity increases exponentially with thickness. However, the value of A of silicone and neoprene materials is relatively small (0.0183 mm-1 and 0.0830 mm-1, respectively), and the exponential growth trend of thermal conductivity is nearly linear.

5. Conclusion

Based on the derivative analysis of Fourier's law, we explain phenomenon of thermal conductivity varying with thickness, which is up to second derivative of temperature. Furthermore, the function of thermal conductivity (FTC) and calculation formula for minimum measured thickness (MMT) are established.

A new parameter (recovery index, A), appearing in the FTC, affects the exponential increasing tendency of the thermal conductivity. Higher the A value is, more obvious the exponential growth trend is, such as GFRP. Otherwise, the exponential growth trend is approximately linear, like neoprene and silicone.

The thermal conductivity is a numerical field. The fitting results of the FTC of FRP, neoprene and silicone at the interface show that the thermal conductivity of the hot strip is different at different powers. This means that the thermal conductivity of any object is not a specific number, but a numerical field.

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