# Equivalent uniaxial constitutive relationship for core concrete of cross-shaped RC short columns

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**Abstract.** Based on the experimental results of seven cross-shaped RC short columns subjected to axial load and reasonable assumptions, the core concrete in cross-shaped section was divided into several regions, the analytic derivation of the effective transverse confined pressure was provided, and the equivalent uniaxial constitutive relationship was constructed for core concrete of the divided regions. The load versus strain curves of the specimens were calculated by the proposed constitutive relationship. The calculated results agreed well with the experimental results. The effects of the tie spacing, diameter, yield stress of hoop reinforcement and section form on lateral confinement were investigated in cross-shaped sections. The results show that the tie spacing, diameter and yield stress of hoop reinforcement has significant influence on axial load versus strain curves. The axial load, peak strain, and strain ductility increase, compared with no lateral confinement of hoop reinforcement. The section form has little influence on axial load versus strain curves.

**Keywords:** Cross-shaped RC short columns; core concrete; equivalent uniaxial constitutive relationship; confined concrete.

## 1. Introduction

In the concrete structures with specially shaped columns, cross-shaped, L-shaped and T-shaped section columns are used instead of ordinary reinforced concrete columns, which have the advantages of rooms not projecting out of the column flutes and increasing the actual use area, etc., and are widely used in China. The force characteristics and seismic performance of specially shaped concrete columns are different from those of ordinary reinforced concrete columns, and their ductility is low. The configuration of the appropriate amount of hoop reinforcement in the specially shaped columns can have a confined effect on the concrete in the core area, effectively limiting the lateral deformation of the concrete, putting the concrete in the triaxial compressive stress state, improving the compressive strength and ductility of the concrete, which is an effective measure to improve the seismic performance of specially shaped concrete columns [1].

At present, some scholars have carried out research on stress-strain models of confined concrete with rectangular, square and circular sections, such as the improved Kent-Park model [2] , Mander model [3] , Saatcioglu model [4] , Chang-Mander model [5] , Légeron-Paultre model [6] , Qian Jiaru model [7] , etc. The study of the uniaxial constitutive relationship of confined concrete with cross-shaped columns is less [8-10] , and it is necessary to study it in depth. In this paper, based on the results of seven cross-shaped RC short concrete columns subjected to axial load in the literature [9], the concrete in the core area of the cross-shaped column section was divided into several regions, and the equivalent uniaxial constitutive relationship of confined concrete in each region was established, which was verified by the test results. At the same time, the influence law of different parameters on the confined effect of hoop reinforcement is analyzed.

## 2. Equivalent uniaxial constitutive relationship of concrete in the core

### 2.1 Confined characteristics of concrete in the core

The confined effect of hoop is mainly applied to the core concrete in the form of "arch effect" [3, 5-6] , under the axial load, the plane where the hoop is located, the horizontal deformation of the core concrete makes the straight section of the hoop bend horizontally, forming an "arch", because the bending stiffness of the hoop is small, the confined effect on the core concrete is weak; while the corner part of the hoop is relatively large, the confined effect on the core concrete is strong. Because of the small bending stiffness of the hoop, the confined effect on the core concrete is weak; while the stiffness of the corner part of the hoop is relatively large, and the confined effect on the core concrete is strong. In each section between adjacent hoops, the effective confined concrete area in the core area varies due to the "arch effect" of hoop restraint, while the effective confined concrete area in the middle section between adjacent hoops is the smallest.

According to the different degrees of hoop restraint, the concrete can be divided into two parts: 1) unconfined zone, i.e., the outer concrete part of the arch; 2) effective confined zone, i.e., the inner concrete part of the unconfined zone, this area of concrete is in a triaxial compressive stress state, and the compressive strength and ductility of the confined concrete are improved. Figure 1 shows the confined partition of cross-shaped concrete column.

The hoop restraint mechanism of cross-shaped column is similar to that of rectangular column, but the confined effect is different, mainly because: 1) the cross-shaped column and rectangular column have different forms of hoop arrangement due to the different section forms, and the confined effect is also different; 2) the section corners of cross-shaped column and rectangular column are different. The cross-shaped column has positive corner and negative corner, while the rectangular column has positive corner, and the confined effect of the hoop reinforcement at positive corner and negative corner on the core concrete is different.

The concrete in the core area of the cross-shaped column is divided into one square and four rectangular regions, as shown in Figure 2, and each region satisfies the longitudinal deformation coordination relationship. Based on the existing research results on the constitutive relationship of confined concrete in rectangular, square and circular columns [2-7], equivalent uniaxial constitutive relationship of concrete in the core area is established for each region separately. For the convenience of expression, it is assumed that the limb height to limb thickness ratio of each column limb is the same in this paper, and the derivation process of the constitutive relationship is similar for unequal limb specially shaped columns.

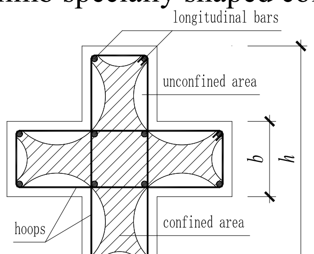


Fig. 1 Regions for unconfined and confined concrete in cross-shaped column

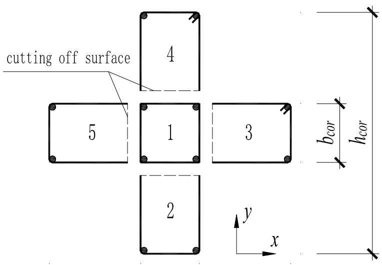


Fig. 2 Divided regions of core concrete in cross-shaped section

## 2.2 Effective transverse confined stress

For the convenience of calculation, the area of confined concrete is generally taken as the area of concrete in the core area. However, because of the "arch effect" of hoop restraint, the effective confined concrete area is smaller than the core concrete area, for this reason, the effective transverse confined stress is calculated according to equation (1) [3].

$$f_l' = k_e f_l \text{ , } k_e = \frac{A_e}{A_{cc}} \tag{1}$$

Where, $f_l$ is the hoop transverse confined stress, $k_e$ is the effective confined factor, $A_e$ is the effective confined concrete area, $A_{cc}$ is the core concrete area (area surrounded by the centerline of the outer hoop).

2.2.1 Effective confined coefficient

In order to simplify and establish a unified form of the expression of the constitutive relationship, the following assumptions are made when calculating the effective confined coefficients in each region: 1) the "arch action" acts on the concrete in the core area in the form of a quadratic parabola with an initial tangent angle of 45° [3, 5-6]; 2) the interface is the effective confined area.

The shapes of the effective confined area and unconfined area of each region of the core concrete in the plane where the hoop reinforcement is located and on the side are shown in Figure 3. In the plane where the hoop reinforcement is located, the area of unconfined area formed by each parabola is $w_j^2/6$, where $w_j$ is the net spacing of adjacent longitudinal reinforcement.

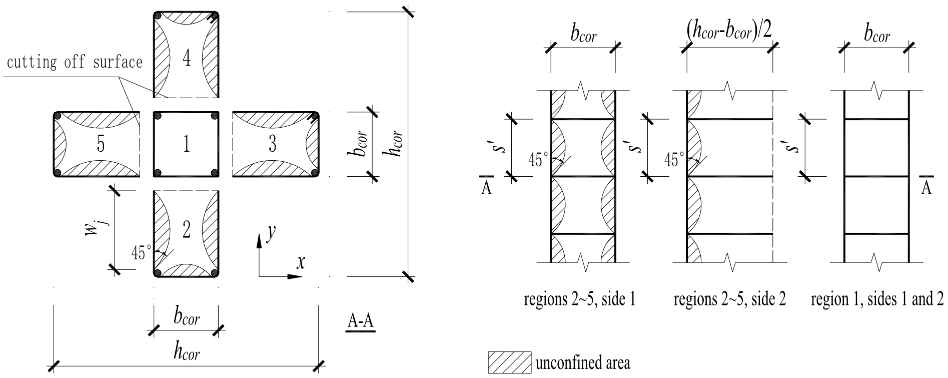



Fig. 3 Effective confined and unconfined core concrete

The total unconfined area of the region $i$ ($i$= 1~5) in the plane where hoops are located is:

$$A_{\mathrm{uh}i}=\sum_{j=1}^{n} w_j^2/6 \tag{2}$$

The total effective confined area of the region $i$ ($i$= 1~5) in the plane where hoops are located is:

$$A_{\mathrm{eh}i}=A_i-A_{\mathrm{uh}i}\,,\ A_i=b_{i1}b_{i2} \tag{3}$$

In the middle section of adjacent hoops, the concrete unconfined area is the largest while the effective confined area is the smallest, and the effective confined concrete area is:

$$A_{\mathrm{e}i}=k_{\mathrm{ez}i}A_{\mathrm{eh}i}\,,\ k_{\mathrm{ez}i}=\frac{\left(b_{i1}-k\times s'/4\right)\left(b_{i2}-m\times s'/4\right)}{A_i} \tag{4}$$

Where, $n$ is the number of unconfined areas in each region, regions 2~5, $n$ = 3, region 1, $n$ = 0; $b_{i1}$, $b_{i2}$ is the width of concrete in the core area of sides 1 and 2 in the region $i$, respectively; $s'$ is the net spacing between hoops; $k$ , $m$ is the number of unconfined areas in the adjacent hoop intermediate section of sides 1 and 2 in regions $i$, regions 2~5, $k$ = 2, $m$ = 1, region 1, $k$ = 0, $m$ = 0.

The effective confined coefficients for the region $i$ ($i$ = 1~ 5) are:

$$k_{\mathrm{e}i}=A_{\mathrm{e}i}/A_i \tag{5}$$

The final effective confined coefficients for each region can be derived as:

$$k_{\mathrm{e1}}=1\,,\ k_{\mathrm{e2}}=k_{\mathrm{e3}}=k_{\mathrm{e4}}=k_{\mathrm{e5}}=\left(1-\sum_{j=1}^{3}\frac{w_j^2}{3b_{\mathrm{cor}}\left(h_{\mathrm{cor}}-b_{\mathrm{cor}}\right)}\right)\left(1-\frac{s'}{2b_{\mathrm{cor}}}\right)\left(1-\frac{s'}{2\left(h_{\mathrm{cor}}-b_{\mathrm{cor}}\right)}\right) \tag{6}$$

Where, $b_{\mathrm{cor}}$, $h_{\mathrm{cor}}$ are the width and height of concrete in the core area of each column limb, respectively, as shown in Figure 3.

For hoop arrangement forms different from that shown in Figure 3, the effective confined coefficients for each region are derived by the same process.

2.2.2 Transverse confined stress of hoop reinforcement

It is assumed that the transverse confined stress in the hoop reinforcement in the core of each region is uniformly distributed, and its inhomogeneity is considered by multiplying the effective confined coefficient of the corresponding region. The length of the separating body with hoop spacing is taken as the study object, and the transverse confined stress of hoop in each region in the direction of $x$ and $y$ can be obtained from the equilibrium condition of force, and it is assumed that the hoop has yielded when the transverse confined stress of hoop reaches the peak [3-5]. The calculation sketch is shown in Fig. 4, and it is obtained that:

$$f_{l2x} = \frac{4 f_{yv} A_{sv1}}{h_{cor} s} , f_{l2y} = \frac{2 f_{yv} A_{sv1}}{b_{cor} s} \tag{7}$$

From the symmetry of the cross section, it follows that:

$$f_{l1x} = f_{l2x} , f_{l1y} = f_{l1x} , f_{l3x} = f_{l2y} , f_{l3y} = f_{l2x} \tag{8}$$

$$f_{l4x} = f_{l2x} , f_{l4y} = f_{l2y} , f_{l5x} = f_{l2y} , \sigma_{l5y} = f_{l2x} \tag{9}$$

Where, $f_{lix}$ and $f_{liy}$ are the transverse confined stresses of hoop reinforcement along $x$ and $y$ respectively, $f_{yv}$ is the yield strength of hoop reinforcement, $A_{sv1}$ is the area of single limb hoop reinforcement, and $s$ is the spacing of hoops.

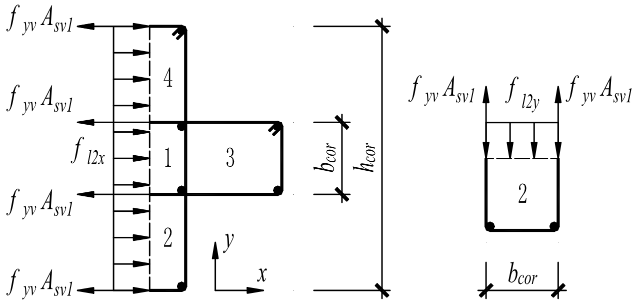


Fig. 4 Calculation sketch of transverse confined stress

For the hoop arrangement form different from that shown in Figure 4, the procedure for deriving the transverse confined stress of hoop reinforcement is the same for each region.

2.2.3 Effective transverse confined stress

From equation (1), it can be seen that the effective transverse confined stresses in the direction of $x$ and $y$ for each region of concrete in the core of are:

$$f'_{lix} = k_{ei} f_{lix} , f'_{liy} = k_{ei} f_{liy} \tag{10}$$

## 2.3 Compressive strength of confined concrete

The effective transverse confined stresses in each region of the core concrete in the $x$ , $y$ directions are not the same, and its compressive strength can be determined by the concrete strength criterion under triaxial stresses. In recent years, some scholars have studied the strength criterion of concrete and established a strength criterion from one parameter to five parameters, such as Lubliner three-parameter strength criterion [11], Hsieh-Ting-Chen four-parameter strength criterion [12], Willam-Warnke five-parameter strength criterion [13], Jiang Mianjing four-parameter strength criterion [14], and Zhenhai-Wang Chuanzhi five-parameter strength criterion [15], etc.

In this paper, the Willam-Warnke five-parameter strength criterion [13] is used to determine the compressive strength of the confined concrete, which can be calculated using equation (11) when the effective transverse confined stresses in both directions are equal [3].

$$f_{cc} = f_c \left( -1.254 + 2.254 \sqrt{1 + \frac{7.94 f'_l}{f_c}} - 2 \frac{f'_l}{f_c} \right) \tag{11}$$

When the effective transverse confined stresses in the direction of $x$ , $y$ are not the same in each region, a simplified equivalent transverse confined stress calculation method is used, the expression of which is as follows:

$$f'_{li} = \frac{f'_{lix} b_{iy} + f'_{liy} b_{ix}}{b_{iy} + b_{ix}} \tag{12}$$

Where, $b_{ix}$ and $b_{iy}$ are the width of the core concrete in the region $i$ along $x$ and $y$, respectively.

Then the compressive strength of the confined concrete in each region is:

$$f_{cc\,i} = f_c\left(-1.254 + 2.254\sqrt{1 + \frac{7.94 f'_{li}}{f_c}} - 2\frac{f'_{li}}{f_c}\right) \tag{13}$$

### 2.4 Peak strain of confined concrete

The hoop confined effect not only increases the compressive strength of concrete, but also increases the peak strain and ductility of concrete. The peak strain of confined concrete in each region is calculated according to equation (14) [3-5].

$$\varepsilon_{cci} = \varepsilon_0\left[1 + 5\left(\frac{f_{cci}}{f_c} - 1\right)\right] \tag{14}$$

Where, $\varepsilon_0$ is the peak strain of unconfined concrete.

### 2.5 Constitutive relationship of confined concrete

According to the confined characteristics of the concrete in the core area of the cross-shaped column, the equivalent uniaxial constitutive relationship of the confined concrete in each region is adopted from the mathematical expression of the Mander model:

$$\sigma_c = \frac{f_{cc}xr}{r-1+x^r},\ x = \frac{\varepsilon_c}{\varepsilon_{cc}},\ r = \frac{E_c}{E_c - E_{sec}},\ E_{sec} = \frac{f_{cc}}{\varepsilon_{cc}} \tag{15}$$

Where, $f_{cc}$, $\varepsilon_{cc}$ are the compressive strength and peak strain of the confined concrete, $\sigma_c$, $\varepsilon_c$ are the stress and strain of the confined concrete, $E_c$ is the elastic modulus of the concrete.

## 3. Proposed constitutive relationship validation

### 3.1 Axial load test of cross-shaped RC short columns

To verify the rationality of the proposed equivalent uniaxial constitutive relationship for concrete in the core area, the axial load test for cross-shaped short RC columns was analyzed theoretically and compared with that in the literature [9]. Seven cross-shaped short column specimens were selected for the test, and the design dimensions and reinforcement configuration of the specimens are shown in Fig. 5. The yield strength and elastic modulus of the longitudinal reinforcement were 307.8 MPa and 2.29×105 MPa, respectively, and the protective layer thickness of the longitudinal concrete was 10 mm. The design parameters of the specimens and the results of the material properties tests are shown in Table 1.

### 3.2 Comparison of analysis results

3.2.1 Comparison of axial compressive bearing capacity and peak deformation

The axial load-deformation curves of the specimens ($P$ - $\varepsilon$) were calculated using the proposed equivalent uniaxial constitutive relationship for the concrete in the core area using the fiber element analysis method. The calculation assumes that: 1) the deformation coordination relationship is satisfied in the longitudinal direction, 2) the stress-strain relationship of the reinforcement is the ideal elastic-plastic constitutive relationship, and 3) the constitutive relationship of the unconfined concrete outside the core is determined according to equation (15), and $f_{cc}$, $\varepsilon_{cc}$ are taken as $f_c$ and $\varepsilon_0$, respectively.

The comparison of experimental results and calculated results are shown in Table 2. The mean and standard deviation of the ratio between the calculated and test values of bearing capacity and peak deformation of the specimens are 1.00 and 0.05 respectively, and the mean and standard deviation of the ratio between the calculated and test values of peak deformation are 0.95 and 0.02 respectively. The established equivalent uniaxial constitutive relationship of concrete in the core area can accurately predict the axial compressive bearing capacity and the corresponding peak deformation of cross-shaped RC short columns.

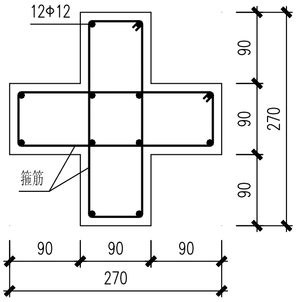



Fig. 5 Design dimensions and reinforcement distribution of specimens

Table 1 Design parameters and material properties of specimens

| Specimen | hoops | $f_c$ / MPa | $f_{yv}$ / MPa | $\varepsilon_0$ / 10-6 | $E_c$ / MPa |
|---|---|---|---|---|---|
| Z+1 | A4@36 | 37.55 | 399.45 | 1914 | 3.43× 104 |
| Z+2 | A4@52 | 34.59 | 399.45 | 1914 | 3.39× 104 |
| Z+3 | A4@68 | 34.59 | 399.45 | 1914 | 3.39× 104 |
| Z+4 | A4@84 | 47.56 | 399.45 | 2012 | 3.57× 104 |
| Z+5 | A6@50 | 47.56 | 383.00 | 2012 | 3.57× 104 |
| Z+6 | A6@62 | 26.00 | 383.00 | 1908 | 3.20× 104 |
| Z+7 | A6@81 | 37.55 | 383.00 | 1914 | 3.43× 104 |

Note: $f_c$ is the axial compressive strength, $E_c$ is the elastic modulus of concrete, $\varepsilon_0$ is the peak strain of concrete，$f_{yv}$ is the yield strength of hoops.

Table 2 Comparison of experimental results and calculated results

| Specimen | $N_{u,t}$ / kN | $\varepsilon_{max,t}$ / 10-6 | $N_{u,c}$ / kN | $\varepsilon_{max,c}$ / 10-6 | $N_{u,d}$ / kN | $\frac{N_{u,c}}{N_{u,d}}$ | $\frac{N_{u,c}}{N_{u,t}}$ | $\frac{\varepsilon_{max,c}}{\varepsilon_{max,t}}$ |
|---|---|---|---|---|---|---|---|---|
| Z+1 | 2150 | 3764 | 2158 | 3689 | 1938 | 1.11 | 1.00 | 0.98 |
| Z+2 | 2000 | 3225 | 1964 | 3043 | 1818 | 1.08 | 0.98 | 0.94 |
| Z+3 | 1780 | 2750 | 1917 | 2704 | 1818 | 1.05 | 1.08 | 0.98 |
| Z+4 | 2335 | 2525 | 2406 | 2435 | 2344 | 1.03 | 1.03 | 0.96 |
| Z+5 | 2885 | 4262 | 2627 | 4017 | 2344 | 1.12 | 0.91 | 0.94 |
| Z+6 | 1750 | 5630 | 1730 | 5377 | 1471 | 1.18 | 0.99 | 0.96 |
| Z+7 | 2120 | 3460 | 2093 | 3166 | 1938 | 1.08 | 0.99 | 0.92 |

Note: $N_{u,t}$ is the test result of bearing capacity, $N_{u,t}$ is the calculated result of bearing capacity, $\varepsilon_{max,t}$ is the test result of peak strain, $\varepsilon_{max,c}$ is the calculated result of peak strain, $N_{u,d}$ is the nominal bearing capacity.

### 3.2.2 Comparison with nominal bearing capacity

According to JGJ149-2017 [16], the nominal bearing capacity of the defined specimen is calculated according to formula (16), which does not consider the hoop confined effect on the increase of concrete strength. The nominal bearing capacity of each specimen, the ratio of the calculated value of bearing capacity to the nominal bearing capacity is shown in Table 2.

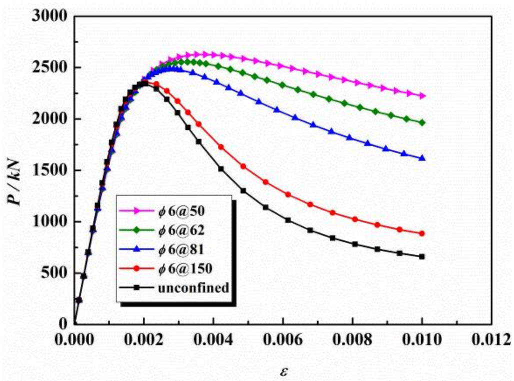
$$N_{u,d} = f_c A + f_y A_s \tag{16}$$

Where, $A$ and $A_s$ are the section area of column and longitudinal reinforcement, respectively, and $f_y$ is the yield strength of longitudinal reinforcement.

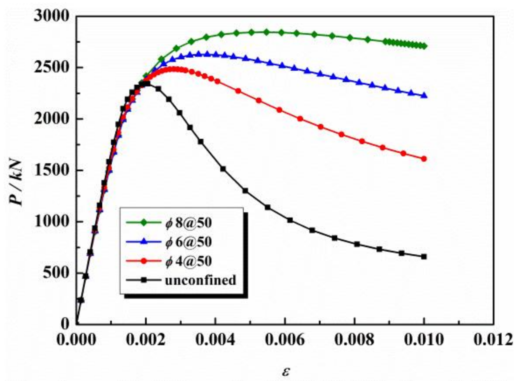
As can be seen from Table 2, the ratio between the calculated value of bearing capacity and the nominal bearing capacity is between 1.03 and 1.18, which indicates that considering the hoop confined effect on the concrete strength, the bearing capacity has increased in different degrees, and the degree of increase is related to the hoop spacing, hoop diameter and hoop yield strength.

### 3.2.3 Influence of different parameters on the confined effect of hoop reinforcement

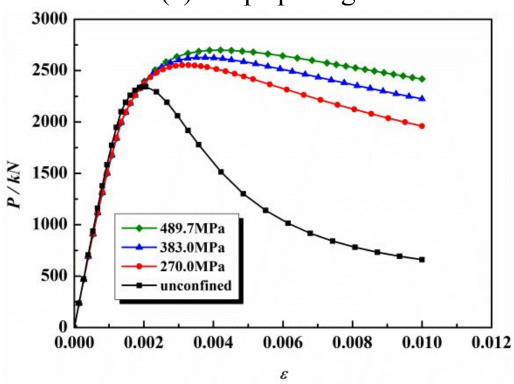
In order to study the influence law of different parameters on the hoop confined effect, the specimen Z+5 is taken as the basic specimen, keeping other parameters unchanged, changing hoop spacing, hoop diameter, hoop yield strength and section form for calculation and analysis, and getting the influence of each parameter on the axial load-deformation curve, as shown in Figure 6.



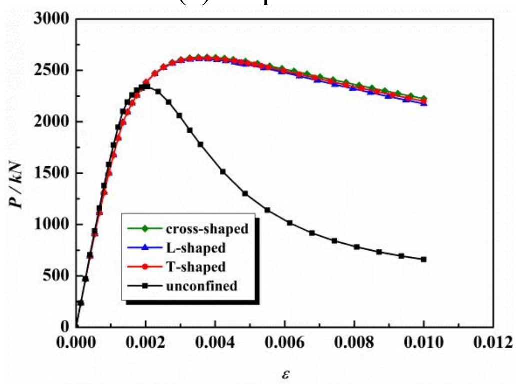

(a) hoop spacing




(b) hoop diameter




(c) hoop yield strength




(d) section form

Fig. 6 Effects of different parameters on axial load- deformation curves

As can be seen from Fig. 6, the hoop spacing, hoop diameter and hoop yield strength have a greater influence on the load-deformation curve, and the bearing capacity, peak deformation and strain ductility are all improved to different degrees compared with the axial load-deformation curve without considering the hoop restraint. With the increase of the hoop spacing, the hoop restraint reduces, and the axial compressive bearing capacity, peak deformation and strain ductility are reduced. When the spacing increases to a certain extent, the load-deformation curve is basically similar to that without considering the confined effect, as shown in Fig. 6a. With the increase of hoop diameter, the hoop restraint is enhanced and the axial compressive bearing capacity, peak deformation and strain ductility are significantly increased, as shown in Fig. 6b. With the decrease of the yield strength of the hoop, the hoop restraint effect decreases, and the axial compressive bearing capacity, peak deformation and strain ductility are all reduced, as shown in Fig. 6c. And the section form has less influence on the load- deformation curve, and the load-deformation curves of the three section forms basically overlap, as shown in Fig. 6d.

# 4. Summary

(1) The concrete in the core area of the cross-shaped column is divided into several regions, the derivation process of the effective transverse confined stress is given, and the equivalent uniaxial constitutive relationship of the concrete in the core area of each region is established.

(2) The load-deformation curves of the specimens from the relevant test were calculated, and the calculated results agreed well with the test results with little dispersion, and the equivalent uniaxial constitutive relationship of concrete in the core area suggested in this paper can accurately predict the axial compressive bearing capacity and the corresponding peak deformation of cross-shaped RC short columns.

(3) Compared with the nominal bearing capacity of the specimens, considering the hoop confined effect on the concrete strength, the bearing capacity are increased in different degrees.

(4) The hoop spacing, hoop diameter and hoop yield strength have greater influence on the axial load-deformation curve, compared with the axial load-deformation curve without considering the hoop restraint, the axial compression bearing capacity, peak deformation and strain ductility are all improved to different degrees. The section form has less influence on the load-deformation curve.

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