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Analysis of Influence of Boundary Points Density on Precision of Sea Plot Area

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Abstract. In order to improve the area accuracy of the sea plot, the formula of the influence of the density of the graphic boundary points on the area accuracy is deduced and stripped. The conditions of the different values of the vertex angle of the boundary points to improve the area accuracy are analyzed separately. The values of the area accuracy of different sea plot under different distribution densities are deduced and simulated. The results show that the more insertion points on the Sea plot line boundary line, the higher the area accuracy. Under the condition that the boundary point accuracy is 0.1m and the error limit of the sea plot area is 100m2, when the area of the Sea plot with a width : length of 1:4 is greater than 9 hm2 and the area of the Sea plot with a width : length of 1:1 is greater than 25 hm2, it is necessary to consider the insertion point to improve the area accuracy. The maximum relative area error of the oil and cable pipeline sea plot is 3.5‰, taking into account the area accuracy of the cable pipeline sea plot and the actual workload, the insertion point interval of 100 ~ 200 m is selected. The formula and quantitative results derived in this paper can be used as a reference for the layout density of sea plot boundary sites.

Keywords:sea plot；area accuracy；boundary point spacing；cable pipeline sea plot；reclamation sea plot.

1. Introduction

The area of the sea plot is an important basic value for the dynamic supervision of the sea area. Accurately grasping the error range of the sea area is an important basis for making accurate regulatory conclusions. At the same time, improving the accuracy of the sea plot area is also an important condition for discovering the potential risks of the sea area.

In the current technical specifications, the area calculation methods of sea plot and parcel (or larger plots) are the same, and both use the horizontal geometric area under projection[1-3], with the same calculation formula. However, there are also differences between the two. First, the parcel is the first target plot, which is used after the determination of the right through the boundary measurement, and the boundary point has a fixed object, while the sea plot is the first to apply for a certain sea area in the space, and the right is used after the approval, and the boundary point has no fixed object ; second, the parcel is generally small, the shape of the figure is irregular, the plot area is generally large, the shape of the figure is more regular. Many related studies have been done on the analysis of the influence of the area accuracy of the parcel (or similar plots). Cheng Shande[4], He Duoxing[5], Guo Zonghe[6], Liu Shaotang[7], Chen Zhirui[8] studied the relationship between the accuracy of the boundary points and the area accuracy. Qi Gongyu[9] studied the influence of the shape of the plot on the area accuracy. Xu Yunyan[10] studied the relationship between the accuracy of the boundary points and the area accuracy, and determined the accuracy requirements of the boundary points to achieve the established goal of the area accuracy. Because the parcel area is small, the expected area accuracy can be achieved by controlling the accuracy of the boundary points.

Because there is no fixed object at the boundary point of sea plot, the marine administrative department will carry out later supervision of sea use. Among them, land reclamation and offshore oil and gas cable pipelines are the most typical, and the application for sea use is the most common and the supervision is the most strict. When the reclamation is completed, it is necessary to carry out the acceptance survey of the completed sea area and carry out the nuclear survey of the sea area ; the same offshore oil and gas cable pipeline sea plot will carry out regular nuclear survey, nuclear survey sea area, nuclear survey results often appear sea plot use area exceeds the approved area. In order to avoid the situation of using the sea beyond the approved range, the research on improving the accuracy of the sea area has been done. Chen Yilan[11] and Zhao Xiaolong[12] studied the use of ellipsoid area instead of projection area to avoid projection error, but this is the replacement of the theoretical truth value, not to reduce the error, especially for the construction of oil and gas pipelines, it is necessary to construct along the direction of the geodetic line to increase the difficulty of construction.

In practical work, the sea range is determined, that is, the shape of the figure is determined, and the space for improving the accuracy of the measuring equipment used is not much. This paper will discuss the method of inserting points on the straight line boundary line. This method is not mentioned in the relevant specifications[2,13], whether it can effectively improve the area accuracy, and quantitatively discuss the influence of the number of insertion points on the area accuracy, that is, the influence of the density of the points on the area accuracy.

2. Analysis of the influence of point density on the accuracy of Sea plot area

An open sea surrounded by n boundary points $(x_i,y_i)(i=1,2,3,...,n)$, its area A is

$$A=\frac{1}{2}\sum_{i=1}^{n}x_i(y_{i+1}-y_{i-1})=\frac{1}{2}\sum_{i=1}^{n}y_i(x_{i-1}-x_{i+1}) \quad (1)$$

which $y_0=y_n$, $y_{n+1}=y_1$, $x_0=x_n$, $x_{n+1}=x_1$.

The area calculation formula on both sides at the same time differential, get,

$$dA=\frac{1}{2}\sum_{i=1}^{n}(y_{i+1}-y_{i-1})\,dx_i+\frac{1}{2}\sum_{i=1}^{n}(x_{i-1}-x_{i+1})\,dy_i \quad (2)$$

According to the law of error propagation, it is also considered that the measurement process of all boundary marks is independent of each other and has no correlation.

$$m_A^2=\frac{1}{4}\sum_{i=1}^{n}(y_{i+1}-y_{i-1})^2m_{x_i}^2+\frac{1}{4}\sum_{i=1}^{n}(x_{i+1}-x_{i-1})^2m_{y_i}^2 \quad (3)$$

In formula (3), m_A is the area accuracy, m_{x_i} and m_{y_i} are the accuracy of the boundary mark i in the x and y directions, which is usually considered , $m_{x_i}^2=m_{y_i}^2=\frac{1}{2}m_P^2$, m_P is the accuracy of the measuring point, then (3) can be changed to

$$m_A=\pm\frac{\sqrt{2}}{4}m_P\sqrt{\sum_{i=1}^{n}[(y_{i+1}-y_{i-1})^2+(x_{i+1}-x_{i-1})^2]}=\pm\frac{\sqrt{2}}{4}m_P\sqrt{\sum_{i=1}^{n}d_{i+1,i-1}^2} \quad (4)$$

$d_{i+1,i-1}$ is the linear distance from point i+1 to point i-1. From Equation (4), the area accuracy is proportional to the accuracy of the boundary marks, and is proportional to the square root of the square sum of the discrete distances with an interval of 1 point. Starting from Equation (4), the influence analysis of the sum of squares of discrete distances will be discussed.

put,

$$\lambda=\sqrt{\sum_{i=1}^{n}d_{i+1,i-1}^2} \quad (5)$$

For a determined sea plot, the number of inflection points is fixed, and the method of reducing $d_{i+1,i-1}^2$ is sought to reduce λ. The variation of λ is analyzed by inserting points on the straight line boundary line. Select the vertex angle of any boundary point in the sea plot polygon (if the convex polygon is the inner angle, if the concave polygon is the outer angle).

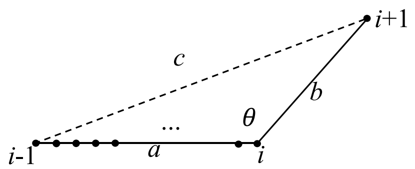

Fig.1 Straight line boundary line insertion point diagram

Fig. 1 is the vertex angle θ corresponding to any boundary mark i. The lengths of two adjacent boundary lines are a and b, respectively. In the triangle surrounded by three boundary points, the length of the opposite side of i is c. Insert m-1 points equidistantly on the boundary line from point i-1 to point i, so that the boundary line is divided into m line segments.

According to Equation (5), the square of the interval distance λ_i^2 at the i-boundary mark is as follows.

$$\lambda_i^2=d_{i+1,i-1}^2 \qquad (6)$$

When there is no insertion point on the boundary line from i-1 to i, according to the law of cosine,

$$\lambda_i^2=c^2=a^2+b^2-2abcos\theta \qquad (7)$$

When m-1 points are inserted on the boundary line from i-1 to i, we get

$$\lambda_i^2=(m-1)(\frac{2a}{m})^2+(\frac{a}{m})^2+b^2-2\frac{ab}{m}cos\theta$$

$$=\frac{(4a^2-2abcos\theta)m-3a^2}{m^2}+b^2 \qquad (8)$$

By observing formula (8), it is concluded that with the increase of m, λ_i^2 must decrease monotonously, and the limit of formula (8) is taken.

$$\lim_{m\to\infty}\lambda_i^2=b^2 \qquad (9)$$

derivative of Equation (8)

$$(\lambda_i^2)'=\frac{[-(4a^2-2abcos\theta)m+6a^2]m}{m^4} \qquad (10)$$

The extreme value is taken when $m=\frac{3a}{2a-bcos\theta}$.

When 2a-bcosθ>0, with $m>\frac{3a}{2a-bcos\theta}$ gradually increases, λ_i^2 will decrease;

When 2a-bcosθ<0, λ_i^2 decreases as m increases. This situation only occurs when θ is an acute angle, according to Equation (7).

$$c^2=a^2+b^2-2abcos\theta<a^2+b^2-4a^2=b^2-3a^2<b^2$$

According to Equation (9), when 2a-bcosθ<0, it is impossible to reduce λ_i^2 by insertion point.

It is also necessary to find a m such that λ_i^2 begins to be equal to c^2 .As m increases, λ_i^2 decreases.

From (8) = c^2,get

$$\frac{(4a^2-2abcos\theta)m-3a^2}{m^2}+b^2=c^2=a^2+b^2-2abcos\theta$$

$$m = \frac{\sqrt{13a^2-4abcos\theta+b^2cos^2\theta}+a+bcos\theta}{4a-2bcos\theta}$$

Therefore, When m is greater than

$$Max(\frac{\sqrt{13a^2-4abcos\theta+b^2cos^2\theta}+a+bcos\theta}{4a-2bcos\theta},\frac{3a}{2a-bcos\theta})$$

, λ_i^2 will decrease.

In summary, the influence of different angle insertion points on λ_i^2 is as follows.

When θ is an obtuse angle, the insertion point will decrease λ_i^2.

When θ is a right angle, λ_i^2 will become larger when one point is inserted. When two points are inserted, λ_i^2 will remain unchanged. When more than two points are inserted, it will gradually become smaller.

When θ is an acute angle, if b > c, the insertion point will only increase λ_i^2, meaningless ; if b < c. After inserting $Max(\frac{\sqrt{13a^2-4abcos\theta+b^2cos^2\theta}+a+bcos\theta}{4a-2bcos\theta},\frac{3a}{2a-bcos\theta})$ points, λ_i^2 decreases.

3. Determination of the density of typical Sea plot boundary points

The actual apex angle of sea plot boundary mark is mostly obtuse angle or right angle, and acute angle will appear in very special cases. Two typical types of sea shape are selected, that is, the rectangle of land reclamation sea and the strip shape of oil and gas pipeline sea plot, to analyze the change of sea area accuracy through insertion point.

3.1 Rectangular sea plot

The rectangular model of sea plot is used to analyze the influence of interpolation points on area accuracy.

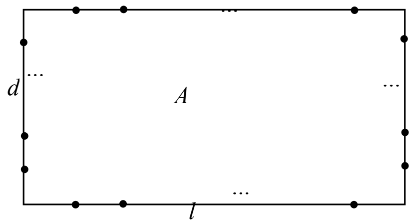

Fig.2 Insertion point diagram of rectangular Sea plot boundary line

Fig.2 is a rectangular sea with a length of l, a width of d and an area of A. The insertion points are equally divided on the length and width. Because the vertices of the boundary are all right angles, k points are inserted equidistantly on each boundary line.

$$\lambda^2=4\frac{2k+1}{(k+1)^2}(l^2+d^2) \quad (11)$$

In Equation (11), $4(l^2+d^2)$ is λ^2 without inserting any point. As the number of insertion points increases, λ^2 gradually decreases.

$$m_A=\pm\frac{\sqrt{2}}{4}m_P\lambda=\pm\frac{\sqrt{2}}{2}m_P\frac{\sqrt{2k+1}}{k+1}\sqrt{l^2+d^2} \quad (12)$$

The formula (12) is the area accuracy formula of sea plot, where $\pm\frac{\sqrt{2}}{2}m_P\sqrt{l^2+d^2}$ are the area accuracy without insertion points. It is concluded that the area accuracy increases with the increase of the number of insertion points.

3.2 Strip-shaped sea plot

In the sea for oil and gas cable pipelines, according to the sea survey specification[13], the sea area is 10 meters outward from the outer edge of the cable pipeline to both sides. Usually the size of the cable pipeline is far less than 20 meters, and the length is more than 1 km. Therefore, the sea area is simplified to a strip shape of 20 meters wide.

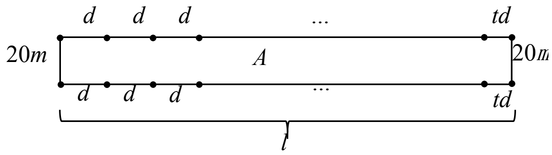

Fig.3 Strip-shaped Sea plot boundary line insertion diagram

Fig. 3 shows a strip-shaped sea plot with a length of l and a width of 20 meters. It is equidistant d insertion point on both sides of the sea plot strip. The last paragraph is denoted by td（$0\leq t\leq 1$）, and there is no insertion point on the 20 meters wide side.

$$\lambda^2=\left[8(\lfloor\rfloor\left(\frac{l}{d}\right)-1)+4(t^2+t+1)\right]d^2+1600, \; d<\frac{l}{2} \quad (13)$$

It is necessary to insert at least two points on the right-angled edge to reduce the conclusion of λ^2, and it is also greater than two points when the model is inserted equidistantly. From Equation (13), as the number of insertion points increases, d gradually becomes smaller, λ^2 becomes smaller, and the area accuracy is improved.

4. Simulation results

According to the sea survey procedures, the accuracy of the boundary points should meet the requirements of ±0.1 meters[13], and m_P=0.1 meters is determined in the following discussion.

4.1 Rectangular sea plot

In the rectangular sea plot , we must discuss d:l according to the formula (11), when d:l is 1:1,1:2,1:3 and 1:4, $\sqrt{l^2+d^2}$ are $\sqrt{2A}$, $\sqrt{\frac{5}{2}A}$, $\sqrt{\frac{10}{3}A}$, $\sqrt{\frac{17}{4}A}$ Select 1 hectare as the basic coefficient calculation, and calculate the change of $|m_A|$ by inserting different points, and get the results of Fig. 4 and Tab. 1.

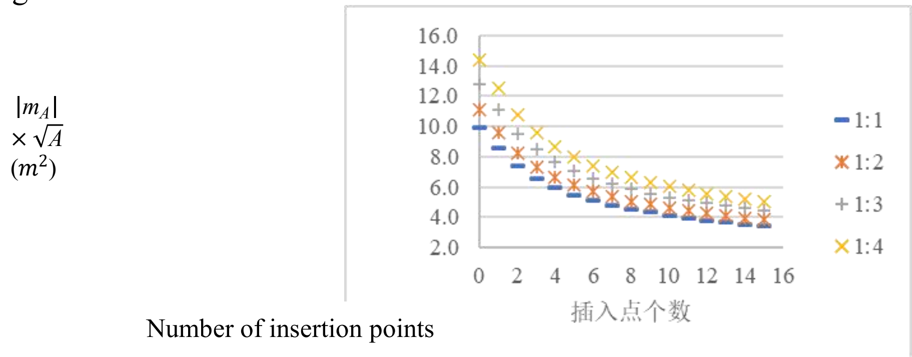

Fig.4 The relationship between the number of interpolation points and the area accuracy under different aspect ratios

It is found from Fig.4 that under the same graphic conditions, the area accuracy increases with the increase of insertion points. In the case of the same insertion point, the width : length closer to 1:1, the higher the area accuracy ; when the graphic conditions are consistent and the number of insertion points is consistent, the larger the area, the lower the accuracy.

Tab. 1 Area accuracy under different width-length ratio, area and number of insertion points
Unit : Area A is hectare, $|m_A|$ is square meter

k	$\|m_A\|$			
	Width : length			
	1:1	1:2	1:3	1:4
0	$9.9\sqrt{A}$	$11.1\sqrt{A}$	$12.8\sqrt{A}$	$14.4\sqrt{A}$
1	$8.6\sqrt{A}$	$9.6\sqrt{A}$	$11.1\sqrt{A}$	$12.5\sqrt{A}$
2	$7.4\sqrt{A}$	$8.2\sqrt{A}$	$9.5\sqrt{A}$	$10.8\sqrt{A}$
3	$6.5\sqrt{A}$	$7.3\sqrt{A}$	$8.5\sqrt{A}$	$9.5\sqrt{A}$
4	$5.9\sqrt{A}$	$6.6\sqrt{A}$	$7.7\sqrt{A}$	$8.7\sqrt{A}$
5	$5.5\sqrt{A}$	$6.1\sqrt{A}$	$7.1\sqrt{A}$	$8.0\sqrt{A}$
6	$5.1\sqrt{A}$	$5.7\sqrt{A}$	$6.6\sqrt{A}$	$7.4\sqrt{A}$
7	$4.8\sqrt{A}$	$5.4\sqrt{A}$	$6.2\sqrt{A}$	$7.0\sqrt{A}$
8	$4.5\sqrt{A}$	$5.1\sqrt{A}$	$5.9\sqrt{A}$	$6.6\sqrt{A}$
9	$4.3\sqrt{A}$	$4.8\sqrt{A}$	$5.6\sqrt{A}$	$6.3\sqrt{A}$
10	$4.1\sqrt{A}$	$4.6\sqrt{A}$	$5.3\sqrt{A}$	$6.0\sqrt{A}$
11	$4.0\sqrt{A}$	$4.4\sqrt{A}$	$5.1\sqrt{A}$	$5.8\sqrt{A}$
12	$3.8\sqrt{A}$	$4.3\sqrt{A}$	$4.9\sqrt{A}$	$5.6\sqrt{A}$
13	$3.7\sqrt{A}$	$4.1\sqrt{A}$	$4.7\sqrt{A}$	$5.4\sqrt{A}$
14	$3.6\sqrt{A}$	$4.0\sqrt{A}$	$4.6\sqrt{A}$	$5.2\sqrt{A}$

15	$3.4\sqrt{A}$	$3.9\sqrt{A}$	$4.4\sqrt{A}$	$5.0\sqrt{A}$

The relative accuracy of the area is obtained by dividing the calculation results in Table 1 by 10000A, and the relative area accuracy increases with the increase of the area. If the relative area accuracy reaches 1%, the required area is less than 0.01 hectares, but the area accuracy value in this case is very small.

Generally, 100 square meters is used as the error limit of sea plot , so m_A should be better than 50 square meters. From Tab. 1, it is concluded that when the area of sea plot with width : length of 1:4 is greater than 9 hectares, the interpolation point should be considered to improve the accuracy ; for the width : length of 1: , when the sea area is greater than 25 hectares, it is necessary to consider the insertion point to improve the area accuracy.

4.2 Strip-shaped sea plot

In the strip-shaped sea plot , the sea range of pipelines with different lengths is discussed. The length of the pipeline is selected as 1 km, 5 km, 10 km and 20 km. According to the formula (12), different interpolation point spacing pairs $|m_A|$ appear the results in Fig. 5 and Tab. 2.

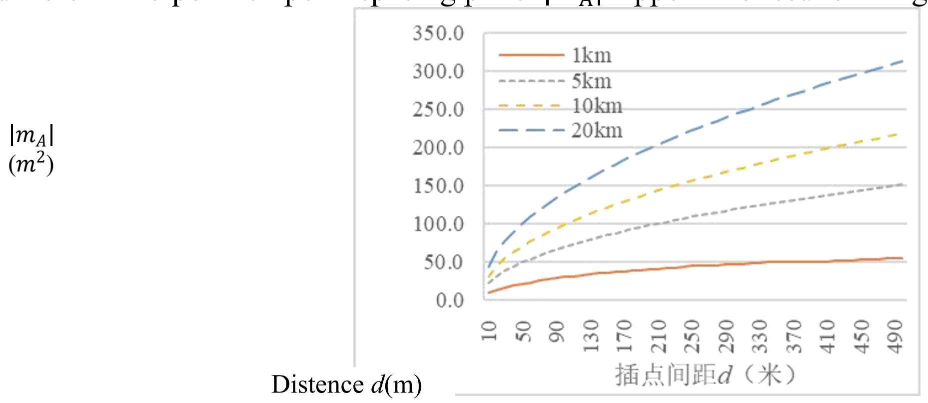

Fig.5 The relationship between the sea area accuracy and the insertion point spacing.

It is found from Fig.5 that in a fixed-length pipeline, the smaller the insertion point spacing (the more insertion points), the higher the area accuracy ; under the same insertion point spacing, the longer the pipeline is, the lower the area accuracy is.

Tab. 2 The area accuracy and relative accuracy of Sea plot under different length and insertion point spacing

Unit : $|m_A|$ is square meter

d(m)	$\|m_A\|$, $\|m_A\|/A$			
	1km	5km	10km	20km
10	10.1,0.5‰	22.4,0.2‰	31.6,0.2‰	44.7,0.1‰
20	14.1,0.7‰	31.6,0.3‰	44.7,0.2‰	63.2,0.2‰
50	22,1.1‰	49.8,0.5‰	70.6,0.4‰	99.9,0.2‰
100	30.4,1.5‰	70.2,0.7‰	99.6,0.5‰	141.2,0.4‰
150	36.2,1.8‰	85.5,0.9‰	121.7,0.6‰	172.7,0.4‰
200	41.3,2.1‰	98.5,1‰	140.4,0.7‰	199.3,0.5‰
No insertion point	70.7,3.5‰	353.6,3.5‰	707.1,3.5‰	1414.2,3.5‰

The relative area accuracy in Tab. 2 increases with the increase of the spacing of the insertion points (the number of insertion points decreases), and the relative area accuracy value increases until the maximum value of 3.5 ‰ when the insertion point is not inserted ; under the same insertion point spacing, the longer the pipeline, the smaller the relative area accuracy.

For the strip-shaped sea, when the error limit of 100 square meters is used, we find that for 20 kilometers of pipelines, it is necessary to increase one point by 10 meters. The workload is too large, and the boundary point positioning accuracy of the sea for oil and gas pipelines is much lower than

0.1m.At this time, it is difficult to apply the absolute amount of $|m_A|$ to practical work. It is possible to select the insertion point spacing in the range of 100-200 meters while taking into account the $|m_A|$ and the workload, so as to control the $|m_A|$ below 50% of the maximum value.

5. Conclusion

Through the method of inserting the boundary mark into the boundary line on the top corner of the boundary mark, it is concluded that the area accuracy will decrease first and then increase with the increase of the insertion point when the apex angle is acute. If it is a right angle, the area accuracy will decrease when one point is inserted, and the area accuracy will increase when the point is added. If the angle is obtuse, as long as the insertion point, the area accuracy can be improved. Therefore, it is necessary to carefully consider adding points when encountering sharp corners in the actual sea.

In typical sea plot graphics, the method of inserting points on the straight line boundary line can improve the area accuracy. The more insertion points, the higher the area accuracy. In the rectangular sea, under the same area, the closer the width : length is to 1:1, the higher the area accuracy is. Under the condition of the same graph, the larger the area, the lower the accuracy of the large area, but the higher the relative area accuracy. In the zonal sea plot, the longer the strip (the larger the area), the lower the area accuracy.

Under the condition that the accuracy of boundary marks meets ±0.1 meters, if the relative area accuracy of rectangular sea plot reaches 1%, the required area is less than 0.01 hectares. The error limit of the rectangular sea area is 100 square meters. When the area of the sea area with width : length of 1:4 is greater than 9 hectares, the interpolation point should be considered to improve the accuracy. When the area of the sea area with width : length of 1:1 is greater than 25 hectares, the interpolation point should be considered to improve the area accuracy. The maximum error of relative area is 3.5‰. The error limit of 100 square meters of sea plot area is not operable for strip sea plot ; considering the area accuracy and workload, the interpolation point spacing is selected in the range of 100 ~ 200 meters.

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