

# Project Scheduling Problems with Constraints on Personnel Scheduling

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**Abstract.** This paper addresses the multifaceted decision-making optimization problem of determining the quantity of in-house and outsourced personnel required for projects, as well as the day-off scheduling for in-house staff, aimed at enhancing project management efficiency. With the backdrop of rapidly changing market demands and the accelerated development of emerging technologies, project-oriented organizations, characterized by their project-centric nature, exhibit high levels of innovation, flexibility, and adaptability, with their influence continually expanding. However, these organizations also encounter challenges related to project downtime, employee retention, and personnel scheduling. Building on the traditional discrete time/resource trade-off problem in project scheduling, this study unpacks the resource component into in-house and outsourced personnel. It also imposes constraints on the consecutive days off and the maximum continuous working hours for in-house staff, thereby establishing an integer programming model. To meet the computational demands of large-scale problems, this paper introduces an Iterated Local Search algorithm for the integrated solution of multi-mode project scheduling and personnel allocation. Through experimental analysis, the effectiveness of the model is demonstrated, and the paper explores the impact of the maximum continuous working hours for staff vacations, and the pricing of in-house and outsourced personnel on decision-making. This provides project-oriented organizations with guidance on conducting project activities, arranging in-house staff vacations rationally, and pricing for internal and external personnel.

**Keywords:** Discrete time/resource trade-off problem; Day-off scheduling; Iterated local search.

## 1. Introduction

Project-based organizations in sectors like construction, IT, and consulting operate around distinct projects, giving project managers autonomy over resource allocation and priorities, as highlighted in the PMBOK Guide's fifth edition. This structure allows for swift adaptation to market shifts and tailored solutions but also poses challenges in managing resources, ensuring revenue stability, and relying on specialized staff. Effective resource management is essential to prevent impacts on project quality and deadlines, while fluctuations in revenue affect strategic planning. The dependency on skilled workers further complicates sustaining a workforce during downtimes.

To address these challenges, firms often keep a minimal permanent workforce, hiring on a project-by-project basis to optimize resource utilization and reduce costs. This paper introduces a quantitative model for efficient project and staff scheduling within tight deadlines, aimed at cost minimization through strategic work mode adjustments and optimal resource allocation. It proposes detailed scheduling for projects and staff to improve resource efficiency and reduce the necessity for extensive in-house personnel, thereby maintaining a sustainable talent pool at a lower cost.

Enhancing traditional approaches to resource management, the model integrates precise resource availability and required rest periods to support innovative task management and effective project delivery. It presents a holistic strategy for project and personnel scheduling, with potential applications in wider operational areas like equipment maintenance. This approach offers project-based companies a strategic framework to overcome their operational challenges, highlighting a path toward more efficient and flexible resource management.

## 2. Organization of the Text

### 2.1 Model building

This paper focuses on minimizing human resource costs within a multi-mode project network and homogeneous human resource requirements, aiming to generate a project schedule and staffing plan. The project schedule encompasses the start times of each activity within the project and the execution modes selected for these activities. The staffing plan details the number of in-house employees, their work-rest patterns, and the daily count of outsourced staff within the project. Initially, a set of vacation patterns and an integer programming model are constructed. The vacation scheduling constraint set specifies the arrangement of workdays and days off, while the integer programming model addresses the project scheduling issue. This includes an objective function for minimizing human resource costs, along with constraints on task logical relationships and human resource limitations. The human resource constraints are informed by the staffing schedule.

#### 2.1.1 Multi-mode project network

The characteristics of activity and project scheduling can be defined as follows: A project is represented by an activity network  $G = (N, A)$ , where the set of nodes  $N$  represents the various activities, and the set of edges  $A$  represents the direct precedence order between activities, as illustrated in Figure. 1 Project Network Diagram .In a multi-mode project, each activity  $i \in N$  is executed according to a specific mode  $m_i$ , which is selected from its corresponding set of modes  $M_i$  Each mode has a fixed duration and requires a certain amount of resources per time unit. For instance, an activity that lasts 5 days with 5 workers versus 8 days with 3 workers are two different execution modes. The schedule of the project is comprised of the start time vector of activities  $S^h = (s_1^h, s_2^h, \dots, s_n^h)$  and the mode vector  $P^h = (p_1^h, p_2^h, \dots, p_n^h)$ .

#### 2.1.2 Personnel scheduling constraints

The demand for human resources generated during the execution of a project can be met by both in-house staff and outsourced personnel. For outsourced personnel, it is assumed that any quantity can be obtained on any day within the project duration. For in-house employees, it is stipulated that they must have two consecutive days off per week, resulting in six possible scheduling patterns per week. These patterns range from staff scheduling mode 1, where the rest days are the first and second days of the week, to staff scheduling mode 6, where the rest days are the sixth and seventh days of the week, as illustrated in Figure. 2 Personnel Scheduling Mode for consecutive rest day scheduling patterns.

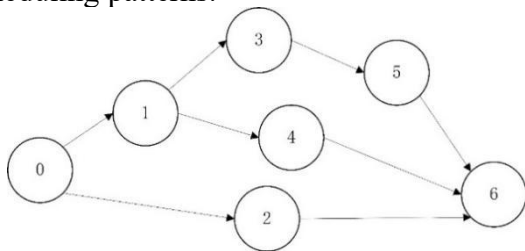


Figure. 1 Project Network Diagram

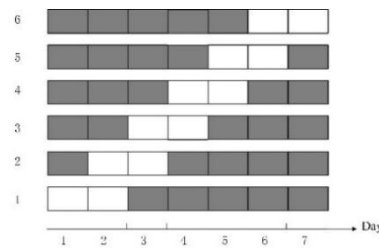


Figure.2 Personnel Scheduling Mode

#### 2.1.3 Model building

Table 1. Project Scheduling Parameters

| Parameters   | Interpretation   |
|--|--|
| $N = \{0, 1, \dots, n + 1\}$                               | task collection  |
| $A = \{(i, j)\}$   | Set of Task Pairs with Precedence Relations              |
| $M = \{M_0, M_1, \dots, M_{n+1}\}$                         | Set of Task Modes  |
| $T_l = \{7 \cdot l - 6, 7 \cdot l - 5, \dots, 7 \cdot l\}$ | Representation of the Project Duration within the l Week |

|  |   |
|--|---|
| $b_{ke} = 1, 0 \quad k \in \{1, \dots, 6\}, t \in \{1, \dots, 7\}$ | $b_{ke} = 1$ indicates that on day $e$ when staff scheduling mode $k$ is adopted, it is a working day; otherwise, it is a rest day. |
| $w$  | Maximum continuous working hours for in-house employees   |
| $r_{im}$   | The number of personnel required per unit time when task $i$ is executed in task mode $m$ .   |
| $d_{im}$   | The duration of task $i$ when executed in task mode $m$ .   |
| $c^{ih}$   | The cost per unit time of in-house employees within the project.  |
| $c^{TP}$   | Cost per unit time for each outsourced personnel.   |

Table 2. Decision variables

| Symbol                | Interpretation   |
|-----------------------|--|
| $x_{im} \in \{0, 1\}$ | If task $i$ is executed in task mode $k$ , then $x_{im} = 1$ ; otherwise, $x_{im} = 0$ |
| $y_{it} \in \{0, 1\}$ | If task $i$ starts at time $t$ , then $y_{it} = 1$ ; otherwise, $y_{it} = 0$           |
| $z_{lk} \in N^+$      | The number of employees adopting schedule mode $k$ in week $l$                         |
| $o_t \in N^+$         | the number of outsourced employees at time $t$   |

**Construct the following mathematical model**

$$\min \quad c \cdot z + \sum_{t \in T} c^{TP} o_t \tag{1}$$

s.t.

$$\sum_{k=1}^6 z_{lk} \cdot b_{k,t-7 \cdot (l-1)} + o_t - \sum_{i \in N} \sum_{m \in M_i} \sum_{i^*=t-d_{im}+1}^t r_{im} x_{im} y_{i^*} \geq 0 \quad \forall t \in T_l, \forall l \in L \tag{2}$$

$$\sum_{k=1}^6 z_{lk} = z \quad \forall l \in L \tag{3}$$

$$\sum_{k=1}^1 z_{l,1} \leq \sum_{k=1}^{w-4} z_{l+1,k} \quad \forall l \in \{1, \dots, |L|-1\} \tag{4}$$

$$\dots \dots \sum_{k=1}^{10-w} z_{l,k} \leq \sum_{k=1}^5 z_{l+1,k}$$

$$\sum_{t \in [EST_i, LST_i]} y_{it} = 1 \quad \forall i \in N \tag{5}$$

$$\sum_{m \in M_i} x_{im} = 1 \quad \forall i \in N \tag{6}$$

$$\sum_{t \in [EST_j, LST_j]} ty_{jt} - \sum_{t \in [EST_i, LST_i]} ty_{it} - \sum_{m \in M_i} d_{im} x_{im} \geq 0 \quad \forall (i, j) \in A \tag{7}$$

$$\sum_{t \in [EST_{n+1}, LST_{n+1}]} ty_{n+1,t} \leq \eta \quad \forall t \in T \tag{8}$$

$$x_{im} = 0, 1 \quad \forall i \in N, \forall m \in M_i \tag{9}$$

$$y_{it} = 0, 1 \quad \forall i \in N, \forall t \in T \tag{10}$$

$$z_{lk} \in N^+ \quad \forall k \in \{1, \dots, 6\} \tag{11}$$

$$o_t \in N^+ \quad \forall t \in T \tag{12}$$

Formula (1) represents the minimization of the cost of in-house and outsourced personnel. Formula (2) indicates that the daily supply of human resources must be greater than or equal to the demand for human resources. Formula (3) states that the number of in-house personnel must remain the same every week. Formula (4) limits the maximum continuous working hours for in-house

employees. Formula (5) specifies that all activities must start within their earliest and latest start times. Formula (6) requires that each activity must select one mode of execution from multiple possible activity modes. Formula (7) indicates that activities with precedence relationships must start the subsequent activity after the completion of the previous one. Formula (8) indicates that the project schedule should meet the project duration<sup>7</sup>.

## 2.2 Solving algorithm

The core of the algorithm in this paper is a local iterative optimization algorithm, with the pseudocode presented in Figure. 3 Iterated local search (ILS). Comparison of test results of calculation examples. The local iterative optimization algorithm is highly sensitive to initial solutions; therefore, this paper employs the following methods to generate a set of initial solutions with relatively good performance. The model relaxes from integer programming to linear programming  $X_{im}, Y_{it} \in (0,1)$ , selecting modes with higher values among multiple task mode choice variables (with a certain probability of mutation), and further determining the start time of tasks and personnel allocation under a fixed task mode. This process is repeated multiple times to generate a set of initial solutions, which are then ranked according to the optimal value of the best solution. The disturbance method of the local iterative optimization algorithm is adjusted to jump from one solution to another within the initial solution set.

## 2.3 Case analysis

This paper considers a standard multi-mode project network with 30 tasks, utilizing instances from the PSPLIB benchmark dataset. It eliminates non-renewable resources from the data, retaining a single type of renewable resource, and forms reasonable pairs of modes (duration and resource demand). We specify the project duration as  $\eta$ , the cost of in-house personnel  $c = c^{ih} \cdot \eta$ . We set  $c^{ih} = 1$ ,  $c^{TP} = 2$ .

The effectiveness of the algorithm is demonstrated by comparing it with direct solutions obtained using Gurobi, as shown in Table. 3 Comparison of test results of calculation examples. Furthermore, this paper investigates the effects of modifications to the constraints on the maximum consecutive working hours in personnel scheduling on the project costs and the deployment of in-house versus outsourced staff. The impact on project costs is quantified by CR, which stands for Cost Ratio. Here, CR is defined as the cost incurred under a specific constraint of maximum continuous working hours divided by the cost when the maximum continuous working hours are set to 10 days, which effectively equates to an unconstrained scenario. The employment patterns of in-house and outsourced personnel are represented by ROI, which is interpreted as the total man-days of outsourced personnel divided by the total quantity of in-house personnel. CR and ROI are shown in Figures. 4 and 5, respectively.

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1:  $s_0 = \text{Generation\_initial\_solution}()$ 
2:  $s^* = \text{local\_search}(s_0)$ 
3: repeat
4:    $s' = \text{Perturbation}(s^*, \text{history})$ 
5:    $s^* = \text{local\_search}(s')$ 
6:    $s^* = \text{Acceptance\_Criterion}(s^*, s^*, \text{history})$ 
7: until termination condition met
    
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Figure. 3 Iterated local search(ILS)

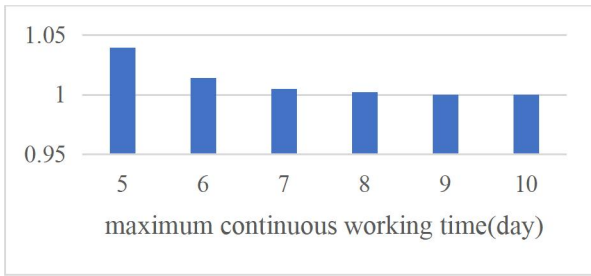


Figure. 4 Cost Ratio(CR)

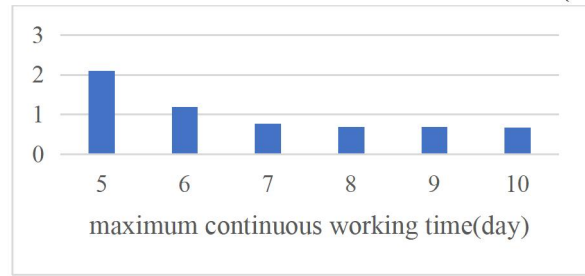


Figure. 5 The ratio of outsourced personnel man-days to the number of in-house personnel

Table 3. Comparison of test results of calculation examples

| Case group | Average total cost |     | Average running time(s) |     |
|------------|--------------------|-----|-------------------------|-----|
|            | Gurobi             | ILS | Gurobi                  | ILS |
| J301_1     | 428                | 429 | 300                     | 769 |
| J301_2     | 834                | 833 | 300                     | 540 |
| J301_3     | 525                | 527 | 300                     | 353 |
| J301_4     | 423                | 424 | 300                     | 450 |
| J301_5     | 513                | 512 | 300                     | 372 |

### 3. Literature References

This paper provides a comprehensive literature review in three main areas: project scheduling, personnel scheduling, and the integrated problem of project scheduling and personnel rostering.

#### 3.1 Discrete time/resource trade-off problem

The Multi-Mode Resource-Constrained Project Scheduling Problem (MRCPSP) represents an extension of the standard Resource-Constrained Project Scheduling Problem (RCPS), focusing on the allocation of the most suitable execution modes for tasks within a project[1]. The concept of MRCPSP, where each task may be executed in multiple modes, each with its distinct resource requirements and durations, stems from the need to address the complexity and flexibility of real-world projects. For instance, in construction projects, a task might be completed using different techniques or teams, with each method impacting the overall project cost and timeline. This notion of multi-modality was originally formalized in the standard models by Van Peteghem and Vanhoucke (2011), Wang and Fang (2011), and Wauters, Verbeeck, Vanden Berghe, and De Causmaecker (2011) [2–4].

An extension of MRCPSP, the discrete time/resource trade-off problem (DTRTP), delves into the optimization of renewable resources only, aiming to balance resource allocation with project duration[5]. Here, the resource constraints are considered soft constraints, allowing for flexibility in project planning. Specifically, Eeckhout and others have focused on a single renewable resource, human resources, with the objective of minimizing the personnel costs required for project execution[6]. This paper builds on these foundations, proposing a computational model to minimize human resource costs within a predetermined project duration, thereby contributing to the field by optimizing cost efficiency in project management.

#### 3.2 Personnel scheduling

Effective personnel scheduling should encompass fair distribution of responsibilities, high employee satisfaction, and efficient resource use[7]. Baker introduced a commonly accepted classification method for personnel scheduling problems, which are primarily categorized into three types: day-off scheduling, shift scheduling, and cyclical scheduling, the latter integrating the first two scheduling challenges[8].

The personnel scheduling scheme used in this article is day-off scheduling. Day-off scheduling, a fundamental and straightforward method of personnel scheduling, focuses on defining specific workdays and days off for each employee. These schedules may vary weekly. In addressing this type of problem, it is assumed that the total number of employees remains constant, with each individual being assigned a plan for workdays and days off.

### 3.3 The integrated Discrete time/resource trade-off problem and Personnel scheduling

In the realm of project scheduling, the scheduling of personnel availability has been less explored. Bailey and colleagues were pioneers in integrating staff vacation considerations into project scheduling, employing homogenized human resources with a singular skill level and adhering to a fixed vacation pattern to minimize labor costs through integer programming[9]. However, this approach determined the personnel schedule based on an already established project timetable, without concurrently generating the project schedule and staff rosters. Alfares and others adopted Bailey's vacation model but expanded to include the scheduling of human resources with varying skill levels[10].

Vanhoucke et al. tackled the scheduling of staff vacations in a multi-project context, treating human resources as shareable across projects. This included adding staff vacation constraints to the project resources and utilizing a single-mode project network[11]. Van Den Eeckhout, Maenhout, and Vanhoucke introduced a variant where labor is the sole renewable resource, incorporating constraints related to staff vacations, such as minimum and maximum consecutive working days, aiming to minimize total labor costs[12].

## 4. Summary

This paper addresses the real-world challenges faced by project-based organizations by proposing a model that balances costs and resources under the constraints of project duration and personnel scheduling strategies. It integrates staffing issues into project scheduling, enriching the project scheduling model, and introduces a computational model based on Iterated local search, demonstrating its effectiveness. The findings offer insights for managers in formulating personnel scheduling policies, such as maximum continuous working hours. Additionally, the model can be applied to large-scale projects, like the Olympics, to calculate the required number of personnel and develop staffing schedules within a defined project timeline.

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