Optimal production decision and supply chain coordination under different capital conditions

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Abstract. This paper investigates a two-level supply chain composed of a retailer and a manufacturer with budget-constrained. The manufacturer's optimal production quantity is explored under different own capital conditions, and the gap between in the decentralized system with wholesale price contract and in the centralized system is discussed. Then, an option contract is used to investigate the supply chain coordination issue. The results show that, when the manufacturer has enough capital, both the production quantity and the expected profit in the decentralized system with wholesale price contract are less than that in the centralized system, and the gap between in the decentralized system and in the centralized system is increasing on the own capital. The option contract can coordinate the supply chain with budget-constrained, and in the presence of supply chain coordination, both the manufacturer and the retailer can become better off by setting reasonable option contracts. Several numerical examples are provided to demonstrate these finds.

Keywords: production decision, supply chain coordination, budget-constrained, option contract.

1. Introduction

Capital is the blood of an enterprise and the basis for its survival and development. In real life, capital is an important factor in enterprise's order or production decision [1], and budget constraint is increasing common in many small and medium-sized enterprises. In particular, budget constraint could be one reason that members order or produce fewer products than the channel-wide optimal-order or optimal-production quantity[2]. That is, as long as any member of a supply chain has a budget constraint, the overall performance of the supply chain may be damaged. Therefore, supply chain coordination is still an important part of supply chain management with budget constraint.

As is well known, there are many studies on supply chain coordination with various contract [3-5]. One of the most popular contracts is the option contract which is common in retailing, toys and communications [6]. In this paper, we first derive the optimal order or production strategy under different own capital conditions, and discuss the expected profits of both the manufacturer and the retailer in the decentralized system with wholesale price contract, as well as the expected profit of the supply chain in the centralized system. Then, we explore how to coordinate the supply chain with budget-constrained by option contract.

2. Assumption

Consider a two-level supply chain consisting of a single manufacturer and a single retailer in one period, and the manufacturer is a small and medium-sized enterprise with budget-constrained. Before the selling period, the manufacturer has only one chance to produce products according to the retailer's order. In addition, the out-of-stock cost is considered in this paper, and in order to simplify the models, we assume that the product has a residual value of 0 at the end of the selling period.

In this paper, we use subscript m for the manufacturer, subscript r for the retailer, and subscript s for the whole supply chain. Simultaneously, we use subscript w for the decentralized system with wholesale price contract, subscript o for the decentralized system with option contract. We let B be the manufacturer's own funds, c be the production cost per unit of product, w be the wholesale price per unit of product, p be the retail price per unit of product and s be the shortage cost per unit of product. The market demand x is a random variable with probability density function $f(x)$ and cumulative distribution function $F(x)$.

3. Benchmark

3.1 Decentralized Decision-Making

In the decentralized system, the wholesale price contract is widely applied in practice. Under the wholesale price contract， the manufacturer set a wholesale price independently or through consultation with the retailer before the start of the selling season, then the retailer decides the quantity of products ordered q_{wr} according to the wholesale price and market demand, where the quantity of products ordered is not greater than the manufacturer's capacity, i.e., $q_{wr} \le B/c$. Finally, the manufacturer produces products according to the retailer's orders.

At the beginning of the selling season, the manufacturer delivers goods to the retailer. After the selling season, if the actual demand is greater than the quantity of products ordered, the retailer bears the loss of stock shortage. And if the actual demand is less than the quantity of products ordered, any unsold products can be salvaged by the retailer.

During the marketing period, the expected sales quantity of product is

$$S(q) = \int_0^q xf(x)dx + \int_q^\infty qf(x)dx \\ = q - \int_0^q F(x)dx \tag{1}$$

At the end of the selling period, the expected surplus quantity of product is

$$I(q_{wr}) = \int_{q_{wr}}^\infty (x - q_{wr})f(x)dx \\ = \mu - q_{wr} + \int_0^{q_{wr}} F(x)dx \tag{2}$$

In the decentralized system, the retailer's problem is expressed as

$$\max_{q_{wr}} \pi_r(q_{wr}) = pS(q_{wr}) - sI(q_{wr}) - wq_{wr} \\ st. \quad q_{wr} \le \bar{q} \tag{3}$$

And the expected profit of the retailer is

$$\pi_r(q_{wr}) = (p - w + s)q_{wr} - s\mu \\ - (p+s)\int_0^{q_{wr}} F(x)dx \tag{4}$$

Since $f(x) > 0$, $p > 0$ and $s > 0$, it is obvious that $\partial^2 \pi_r(q_{wr}) / \partial(q_{wr})^2 = -(p+s)f(q_{wr}) < 0$, that is, $\pi_r(q_{wr})$ is a concave in q_{wr}. Then, the Lagrange method is used further for solution. Let $g(q_{wr}) = \bar{q} - q_{wr}$, we have

$$\begin{cases} \dfrac{\partial \pi_r(q_{wr})}{\partial q_{wr}} + \lambda \dfrac{\partial g(q_{wr})}{\partial q_{wr}} = 0 \\ \lambda\left(\bar{q} - q_{wr}\right) = 0 \end{cases} \tag{5}$$

When $\lambda = 0$, we have $q_{wr}^{*} = F^{-1}((p-w+s)/(p+s))$. For convenience of expression, let $\overline{q_{wr}} = F^{-1}((p-w+s)/(p+s))$, we can get $q_{wr}^{*} = \overline{q_{wr}}$.

When $\lambda = (p-c+s)-(p+s)F(q_{wr}) > 0$, we get $q_{wr}^{*} = \bar{q}$. Namely, when $\bar{q} < \overline{q_{wr}}$, we have $q_{wr}^{*} = \bar{q}$. Thus, the optimal order quantity of the retailer is expressed as

$$q_{wr}^{*} = \begin{cases} \overline{q_{wr}} & \bar{q} \geq \overline{q_{wr}} \\ \bar{q} & \bar{q} < \overline{q_{wr}} \end{cases} \tag{6}$$

The manufacturer uses the “make-to-order” production policy, and it’s expected profit is

$$\pi_{wm}(q_{wr}^{*}) = (w-c)q_{wr}^{*} \tag{7}$$

In sum, the expected profit of the whole supply chain in the decentralized system is

$$\begin{aligned} \pi_s(q_{wr}^{*}) &= \pi_r(q_{wr}^{*}) + \pi_m(q_{wr}^{*}) \\ &= (p-c+s)q_{wr}^{*} - s\mu \\ &\quad -(p+s)\int_0^{q_{wr}^{*}} F(x)dx \end{aligned} \tag{8}$$

3.2 Centralized Decision-Making

In the centralized system, the retailer owns a manufacturer and controls the whole supply chain, and the objective is to maximize the expected profit of the supply chain by deciding the order/production quantity q_s. Thus, the problem can be expressed as

$$\begin{aligned} &\max_{q_s} \pi_s(q_s) = pS(q_s) - sI(q_s) - cq_s \\ &st. \quad q_s \leq \bar{q} \end{aligned} \tag{9}$$

Since $\partial^2 \pi_s(q_s)/\partial(q_s)^2 = -(p+s)f(q_s) < 0$, we can get that $\pi_s(q_s)$ is a concave in q_s. Using the Lagrange method similarly, the optimal order/production quantity of the supply chain in the centralized system is

$$q_s^{*} = \begin{cases} \overline{q_s} & \bar{q} \geq \overline{q_s} \\ \bar{q} & \bar{q} < \overline{q_s} \end{cases} \tag{10}$$

where $\overline{q_s} = F^{-1}((p-c+s)/(p+s))$.

Therefore, the expected profit of the whole supply chain in the centralized system is

$$\pi_s(q_s^*)=(p-c+s)q_s^*-s\mu \\ -(p+s)\int_0^{q_s^*}F(x)dx \tag{11}$$

Note that, we can easily get $\overline{q_s}>\overline{q_{wr}}$ since $w>c$. That is, when the manufacturer has sufficient capital, i.e., B satisfies $\bar{q}\geq\overline{q_s}$, the expected profit of the whole supply chain in the decentralized system is obviously less than that in the centralized system.

When B satisfies $\bar{q}\leq\overline{q_{wr}}$, we have $q_{wr}^*=\bar{q}$. the optimal order/production quantity of the supply chain in the centralized system is determined by $q_s^*=\bar{q}$ because of the limited capital, and the optimal policies both in the decentralized system and in the centralized system are consistent.

When B satisfies $\overline{q_{wr}}<\bar{q}<\overline{q_s}$, the supply chain in the centralized system can use all its own funds for purchasing or production, namely, we have $q_s^*=\bar{q}$. The optimal order quantity of the retailer in the decentralized system is determined by $q_{wr}^*=\overline{q_{wr}}$. Thus, the expected profit of the whole supply chain in the centralized system is obviously more than that in the decentralized system.

4. Supply Chain Coordintion

When and only when B satisfies $\bar{q}\leq\overline{q_{wr}}$, the optimal policies both in the decentralized system and in the centralized system are consistent. In this section, an option contract is introduced to explore the production decision and supply chain coordination issues when B satisfies $\bar{q}>\overline{q_{wr}}$. Under option contract, the sequence of events is described as follows. Before the selling season, the retailer offers an option contract (o,e) to the manufacturer through negotiation, the manufacturer decides the production quantity q_{om} based on the option contract and its own funds, where $q_{om}\leq\bar{q}$. Finally, the retailer pays full option fee oq_{om} to the manufacturer.

According to the option contract, the retailer no longer needs to order products, and the manufacturer takes inventory risk. At the beginning of the selling season, market demand is observed, the retailer may purchase products from the manufacturer at the exercise price to meet market demand during the selling season, where the maximum possible quantity of products purchased is the manufacturer’ s production quantity. If the actual market demand is greater than the manufacturer’ s production quantity, the manufacturer bears the loss of stock shortage. And if the actual market demand is less than the manufacturer’ s production quantity, any unsold products can be salvaged by the manufacturer.

Therefore, under the option contract, the problem of the manufacturer is expressed as

$$\max_{q_{om}}\pi_{om}=E[eS(q_{om})+(o-c)q_{om}] \\ \text{st.}\quad q_{om}\leq\bar{q} \tag{12}$$

The expected profit of the manufacturer under the option contract is given by

$$\pi_{om}(q_{om})=(o+e-c)q_{om}-e\int_0^{q_{om}}F(x)dx \tag{13}$$

We can easily get that $\partial^2\pi_{om}(q_{om})/\partial(q_{om})^2 = -ef(q_{om}) < 0$, namely, $\pi_{om}(q_{om})$ is a concave function of q_{om} . Using the Lagrange method similarly, the optimal production quantity of the manufacturer under option contract is

$$q_{om}^* = \begin{cases} \overline{q_{om}} & \overline{q_{om}} \le \bar{q} \\ \bar{q} & \overline{q_{om}} > \bar{q} \end{cases} \tag{14}$$

where $\overline{q_{om}} = F^{-1}((o+e-c)/e)$.

In this case, the retailer's expected profit can be given by

$$\pi_{or}(q_{om}^*) = (p-e)S(q_{om}^*) - oq_{om}^* - sI(q_{om}^*) \tag{15}$$

Further, the retailer's expected profit can be rewritten as

$$\pi_{or}(q_{om}^*) = (p-o-e+s)q_{om}^* - s\mu - (p-e+s)\int_0^{q_{om}^*} F(x)dx \tag{16}$$

In sum, the expected profit of the whole supply chain under option contract is

$$\pi_{os}(q_{om}^*) = \pi_{om}(q_{om}^*) + \pi_{or}(q_{om}^*) = \pi_s(q_{om}^*) \tag{17}$$

In general, due to the principle of individual rationality, the optimal production quantity of the manufacturer with the option contract in the decentralized system is not greater than the optimal order/production quantity of the supply chain in the centralized system. Namely, we have $\overline{q_{om}} \le \overline{q_s}$.

When the manufacturer has sufficient capital, i.e., $\bar{q} \ge \overline{q_s}$, the optimal order/production quantity of the supply chain in the centralized system is given by $q_s^* = \overline{q_s}$, and the optimal production quantity of the manufacturer in the centralized system is given by $q_{om}^* = \overline{q_{om}}$. When the option contract provided by the retailer satisfies $\overline{q_{om}} = \overline{q_s}$, the supply chain can be coordinated, and the optimal policies both in the decentralized system and in the centralized system are consistent. That is, $F^{-1}((o+e-c)/e) = F^{-1}((p-c+s)/(p+s))$. Since $F(x)$ is a monotonically increasing function, its inverse function $F^{-1}(x)$ is also a monotonically increasing function, thus we get $(o+e-c)/e = (p-c+s)/(p+s)$.

Remark 1: When $\bar{q} \ge \overline{q_s}$ and the option contract (o,e) satisfies $(o+e-c)/e = (p-c+s)/(p+s)$, the supply chain can be coordinated.

When the manufacturer's own funds cannot achieve the optimal production quantity of the supply chain without budget-constrained in centralized system, i.e., $\overline{q_{om}} < \bar{q} < \overline{q_s}$, the optimal order/production quantity of the supply chain in the centralized system is given by $q_s^* = \bar{q}$, and the optimal production quantity of the manufacturer in the decentralized system with the option contract is still given by $q_{om}^* = \overline{q_{om}}$. When the option contract provided by the retailer satisfies $\overline{q_{om}} = \bar{q}$, the optimal policies both in the decentralized system and in the centralized system are consistent. Thus, we have $F^{-1}((o+e-c)/e) = B/c$.

Remark 2: When $\overline{q_{om}} < \bar{q} < \overline{q_s}$ and the option contract (o,e) satisfies $F^{-1}((o+e-c)/e) = B/c$, the supply chain can be coordinated.

When the manufacturer's optimal production quantity under the option contract is greater than the maximum quantity of products produced by the manufacturer with own capital, i.e., $\overline{q_{wr}} < \bar{q} < \overline{q_{om}}$, the optimal order/production quantity of the supply chain in the centralized system is equal to the optimal production quantity of the manufacturer in the decentralized system, and we have $q_{om}^* = q_s^* = \bar{q}$. That is, when $\overline{q_{wr}} < \bar{q} < \overline{q_{om}}$, the supply chain can be still coordinated by adjusting the parameters of the option contract.

5. Numerical Analysis

In this section, we use several numerical experiments to demonstrate our finds. For the sake of simplicity, the market demand is usually assumed to follow uniform distribution [7]. Here, we assume that the stochastic demand x follows uniform distribution with $F(x) \sim \mathrm{U}(0,500)$, the other parameters are as follows: $c = 20$, $w = 40$, $p = 50$, $s = 25$.

Figure 1 shows the optimal production decisions of the manufacturer in the decentralized system with wholesale price contract and in the centralized system. When the own funds of the manufacturer is less than or equal to 5000, the optimal production quantity in the decentralized system with wholesale price contract is consistent with that in the centralized system, and is strictly increasing on the manufacturer's own funds. When the own funds of the manufacturer is greater than 5000 and less than 7500, the optimal production quantity in the decentralized system with wholesale price contract remains unchanged, while the optimal production quantity in the centralized system is strictly increasing on the manufacturer's own funds. Consequently, the gap of the optimal production quantity between in the decentralized system with wholesale price contract and in the centralized system is strictly increasing on the manufacturer's own funds. When the own funds of the manufacturer is greater than 7500, the optimal production quantities both in the decentralized system with wholesale price contract and in the centralized system remain unchanged.

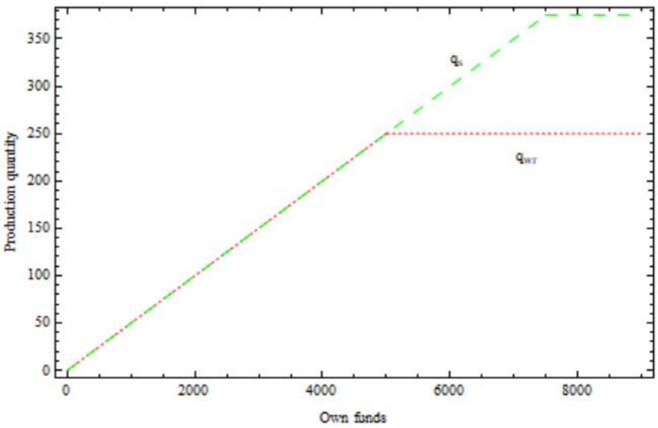

Figure 1 Optimal production decisions in the decentralized sytem with wholesale price contract and in the centralized system.

Figure 2 shows the expected profit changes of the manufacturer, the retailer and the whole supply chain with respect to the manufacturer's own funds. When the own funds of the manufacturer is less than 5000, the expected profits of the manufacturer, the retailer and the whole supply chain in the decentralized system with wholesale price contract are strictly increasing on the manufacturer's own funds. Also, the expected profit of the supply chain in the centralized system is strictly increasing on the manufacturer's own funds. When the own funds of the manufacturer is greater than 5000, all the expected profits of the manufacturer, the retailer and the whole supply chain in the decentralized system with wholesale price contract remain unchanged, while the expected profit of the supply chain in the centralized system is still strictly increasing on the manufacturer's own funds when the own funds of the manufacturer is less than 7500. In addition, when the own funds of the manufacturer is greater than 7500, the expected profit of the supply chain in the centralized system also remains unchanged.

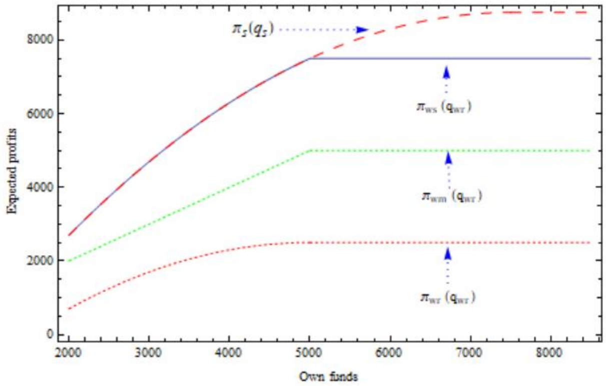

Figure 2 Expected profits in the decentralized sytem with wholesale price contract and in the centralized system.

Under supply chain coordination with option contract, we can see from Figure 3 that the exercise price is strictly decreasing on the option price when the own funds of the manufacturer is given. And given the option price, the greater the own funds of the manufacturer, the greater the exercise price. That's because if the manufacturer has enough money, the retailer will set a higher exercise price to induce the manufacturer to produce more products.

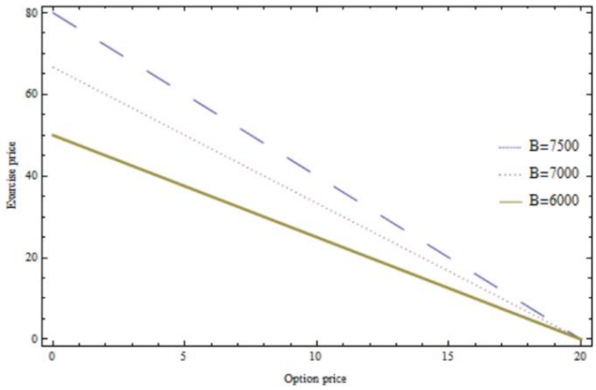

Figure 3 The impact of the option price on the exercise price under different terms of own funds.

Under supply chain coordination with option contract, Figure 4 depicts the expected profit changes of both the manufacturer and the retailer with respect to the option price. With the increase in the option price, the expected profit of the manufacturer decreases, while the retailer's expected profit increases with the increase in the option price. That is, both the manufacturer and the retailer can become better off by setting reasonable option contracts.

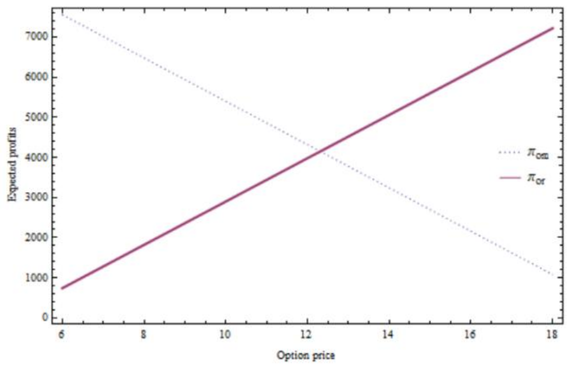

Figure 4 Expected profit changes of both the manufacturer and the retailer with respect to the option price.

6. Conclusions

In this paper, we have studied a two-level supply chain with budget-constrained and investigated the optimal order or production decisions. Also, we analyze how the manufacturer’ s own funds affects the optimal order or production decisions. In addition, option contract is introduced to explore supply chain coordination. This study provides decision and strategy reference for production optimization and supply chain coordination with budget-constrained.

The main result of this paper is summarized as follows. When the manufacturer has less capital, the optimal policies both in the decentralized system and in the centralized system are consistent. When the manufacturer has enough capital, both the production quantity and the expected profit of the whole supply chain in the decentralized system with wholesale price contract are less than that in the centralized system, and the gap between in the decentralized system and in the centralized system is increasing on the manufacturer’ s own capital. The option contract can coordinate the supply chain with budget-constrained, and the exercise price is decreasing in the option price, while increasing in the manufacturer’ s own capital under supply chain coordination. In the presence of supply chain coordination, the expected profit of the manufacturer is decreasing on the option price, while the retailer’ s expected profit is increasing on the option price. So both the manufacturer and the retailer can become better off by setting reasonable option contracts.

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