The Subexponential Distribution wthin Renewal Model

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Abstract. Here, we investigate asymptotic equivalent formulas for finite-time ruin probabilities in a renewal risk model with a subexponential distribution in this research, in the cases of the negatively dependent and upper tail independent, respectively. First, we extend the related research results of Tao Jiang, and Chengguo Weng et al. to the subexponential case, respectively. Thereafter two precise asymptotic equivalent relations are established, one is the finite-time probability when the random variables of the claims are influenced by the common subexponential distribution in a negative way, and another the finite-time ruin probability when the claimsizes are upper tail independent with the common subexponential distribution.

Keywords: The renewal risk model; Subexponential distribution; Negatively dependent; Upper tail independent

1.   Introduction

The research of heavy-tailed distribution is an important direction of the risk theory. When the claim is heavy-tailed distribution, asymptotic estimate theory has been widely studied and developed since the Cramér-Lundberg theory was established. In order to establish the equivalent relation under the assumption of heavy tailed distribution, in different circumstances, Many scholars have achieved fruitful results, such as Hao and Tang[1] studied a renewal model comparable formula with a subexponential tail of discounted aggregate claims, the asymptotic expression for the probability of the large claims within renewal risk model with randomly heavy-tailed delay is given by Wang et al. [2] (refer to[3]), Yang et al. [4] further showed that there was a certain dependent relationship between the random variable and the random weight, where asymptotic equivalent formula of the maximum was given, and so on. Based on predecessors' work, in this research, we investigate two forms of equivalence connections between the negatively dependent and upper tail independent subexponential distributions.

2. Risk Model

In this research, if we add the constant interest force and renewal theory to the classical risk model, we can get

$$\sigma_n = \sum_{i=1}^{n} \theta_i \; , \quad N(t) = \sup\{n > 1 : \sigma_n \le t\} \; , \quad m(t) = E[N(t)] = \sum_{n=1}^{\infty} \Pr(\sigma_n \le t) \; , \; t \ge 0 \; .$$

It should be pointed out that the claim $\{X_n, n \ge 1\}$ is common distribution, while it is also dependent non-negative random variables series. The mean $\mu = EX_1$ is finite. The time interval $\{\theta_i, i \ge 1\}$ of the arrival of the claim is another independent, identically distributed and non-negative random variables series. Moreover, $\{X_n, n \ge 1\}$ and $\{\theta_i, i \ge 1\}$ are mutual independence. Then, using a common constant interest force, the classical risk model can be enhanced to a renewal risk model:

$$U_\delta(t) = u e^{\delta t} + c \int_0^t e^{\delta(t-y)} dy - \sum_{i=1}^{N(t)} X_i e^{\delta(t-\sigma_i)} \; , \tag{1}$$

where u is the initial capital, and c is premium rate which is represented for the premium charged in the unit time, and δ is constant interest force which is correspond to the interest rate and discount rate. The finite time ruin probability can be described as

$$\psi(u, T) = \Pr(U(t) < 0, 0 \leq t \leq T \mid U(0) = u) \qquad (2)$$

The research above is based on the assumption that the claim is distributed independently and uniformly. Next we study asymptotic expression of the probability under the heavy-tailed distribution following negatively dependent, and upper tail independent, respectively, in the case of subexponential distribution.

Definition 1. Definitions of Negatively dependent and Pairwise negatively dependent(See reference [5]).

If claims X_i have a common distribution and are negatively dependent random variables, with a constant interest force, we have a negatively dependent renewal risk model:

$$U_\delta(t) = ue^{\delta t} + c\int_0^t e^{\delta(t-y)}dy - \sum_{i=1}^{N(t)} X_i e^{\delta(t-\sigma_i)} \qquad (3)$$

In the model(3), Chen and Ng. [6]considered that when claims $F \in \mathcal{ERV}(-\alpha, -\beta)$, the probability is:

$$\psi(u, \ T) \sim \int_0^T \overline{F}(ue^{\delta t})dm(t) \qquad (4)$$

Wang et al. [2] considered the ruin probability under the circumstances of $F \in \mathcal{C}$, and Tao[3] extended it to $F \in \mathcal{L} \cap \mathcal{D}$, the relation of probability (4) still holds. In the following, we will investigate the case $F \in \mathcal{S}$.

Definition 2[7]. If the contiguous function C(Coupla): $\lambda = \lim_{v \to 1^-} \dfrac{1 - 2v + C(v,v)}{1-v}$ exists when $\lambda \in (0,1]$, the contiguous function C is upper tail dependent; when $\lambda = 0$, the contiguous function C is upper tail independent.

According to Sklar's Theorem, upper tail independent can be equivalent to: if $\{X_i, i \geq 1\}$ follows common distribution and satisfy,

$$\lim_{x \to \infty} \frac{\Pr(X_i > x, X_j > x)}{\Pr(X_i > x)} = 0, i \neq j, i \geq 1, j \geq 1 \qquad (5)$$

we call $\{X_i, i \geq 1\}$ for binary upper tail independent[8].

Remark 1 It follows from (5) that

$$0 \leq \frac{\Pr(X_j e^{-\delta\sigma_j} > x, X_l e^{-\delta\sigma_l} > x)}{\overline{F}(x)} \leq \frac{\Pr(X_j e^{-\delta\sigma_1} > x, X_l > x)}{\overline{F}(x)}$$

$$\leq \frac{\Pr(X_j > x, X_l > x)}{\overline{F}(x)} \to 0$$

Therefore,

$$\lim_{x \to \infty} \frac{\Pr(X_j e^{-\delta\sigma_j} > x, X_l e^{-\delta\sigma_l} > x)}{\overline{F}(x)} = 0 \qquad (6)$$

Remark 2 If the random variable series $\{X_i, i \geq 1\}$ are independent and identical distribution, (5) is still satisfied, i.e. it satisfies upper tail independent. If the random variable series $\{X_i, i \geq 1\}$ are negatively dependent, it comes

$$0 \leq \lim_{x \to \infty} \frac{\Pr(X_i > x, X_j > x)}{\Pr(X_i > x)} \leq \lim_{x \to \infty} \frac{\Pr(X_i > x)\Pr(X_j > x)}{\Pr(X_i > x)} = 0, \ i \neq j, i \geq 1, j \geq 1.$$

Preliminaries

Lemma 1 [9, 10]Let X and $\{Y, Y_j, j \geq 1\}$ be mutual independence, and X, Y_j and XY respectively follow the distribution F, H and G.

(1)If $F \in \mathcal{S}$, it comes $\lim_{x \to \infty} \dfrac{e^{-\varepsilon x}}{\overline{F}(x)} = 0$ for $\forall \varepsilon > 0$.

(2)If $F \in \mathcal{S}$ and $\overline{H}(x) = o(\overline{F}(x))$, it comes $F * H \in \mathcal{S}$ and $\overline{F * H}(x) \sim \overline{F}(x) + \overline{H}(x) \sim \overline{F}(x)$.

If there exists a constant $c > 0$, such that $\overline{H}(x) \sim c\overline{F}(x)$, it obtains $H \in \mathcal{S}$, $F * H \in \mathcal{S}$ and $\overline{F * H}(x) \sim (1+c)\overline{F}(x)$.

If $F \in \mathcal{S}$, and $\overline{H}(x / a) = o(\overline{G}(x))$ for any $a > 0$, $XY \in \mathcal{S}$ and $X \prod_{j=1}^{n} Y_j \in \mathcal{S}$.

(3) If $F \in \mathcal{L}$ and H is non-degenerate into 0, and $\overline{G}(x / a) \sim o(\overline{H}(x))$ for any given $a > 0$, we can get $H \in \mathcal{L}$.

Lemma 2 [3]In the renewal risk model with constant interest force, claims $\{X_i, i \geq 1\}$ are pairwise negatively dependent random variable series, which follow common distribution F, and $F \in \mathcal{L} \cap \mathcal{D}$, then for any $m_0 > 0$,

$$\Pr(\sum_{i=1}^{m_0} X_i e^{-\delta \sigma_i} > x) \sim \sum_{i=1}^{m_0} \Pr(X_i e^{-\delta \sigma_i} > x) \tag{7}$$

We extend Lemma 2 to subexponential distribution case below.

Corollary 1 We change condition $F \in \mathcal{L} \cap \mathcal{D}$ in lemma 2 to condition $F \in \mathcal{S}$, and lemma 2 is also true.

Proof. Denote $Y_i = X_i e^{-\delta \sigma_i} 1_{(\sigma_i < t)}$. The risk function is $H_i(x)$.

Since

$$\Pr(\sum_{i=1}^{m_0} X_i e^{-\delta \sigma_i} > x) = \overline{H_1 * H_2 * \cdots * H_{m_0}}(x) \quad \sum_{i=1}^{m_0} \Pr(X_i e^{-\delta \sigma_i} > x) = \sum_{i=1}^{m_0} \overline{H_i}(x),$$

we need to prove

$$\overline{H_1 * H_2 \cdots * H_{m_0}}(x) \sim \sum_{i=1}^{m_0} \overline{H_i}(x) \tag{8}$$

When $m_0 = 1$, it is obvious that (4) holds. Assuming that $m_0 \geq 2$, clearly,

$$\Pr(\sum_{i=1}^{m_0} X_i e^{-\delta \sigma_i} > x) \geq \Pr(\bigcup_{i=1}^{m_0} (X_i e^{-\delta \sigma_i} > x))$$

$$\geq \sum_{i=1}^{m_0} \Pr(X_i e^{-\delta \sigma_i} > x) - \sum_{1 \leq j \neq l \leq m_0} \Pr(X_j e^{-\delta \sigma_j} > x, X_l e^{-\delta \sigma_l} > x)$$

When $1 \leq j \neq l \leq m_0$, according to the theory of negatively dependent,

$$\Pr(X_j e^{-\delta \sigma_j} > x, X_l e^{-\delta \sigma_l} > x) \leq \Pr(X_j e^{-\delta \sigma_1} > x, X_l > x)$$
$$\leq \Pr(X_j e^{-\delta \sigma_1} > x) \Pr(X_l > x) = o(\Pr(X_1 e^{-\delta \sigma_1} > x)).$$

We can get

$$\Pr(\sum_{i=1}^{m_0} X_i e^{-\delta \sigma_i} > x) \geq \sum_{i=1}^{m_0} \Pr(X_i e^{-\delta \sigma_i} > x) \tag{9}$$

For any constant m,

$$\Pr(\sum_{i=1}^{m_0} X_i e^{-\delta \sigma_i} > x) \leq$$
$$\Pr(\bigcup_{1 \leq i \leq m_0} Y_i > x - m) + \Pr(\sum_{i=1}^{m_0} Y_i > x, \bigcap_{1 \leq i \leq m_0} Y_i \leq x - m)$$
$$= I_1(x, m) + I_2(x, m)$$

According to Lemma 1,

$$I_1(x, m) \leq \sum_{i=1}^{m_0} \Pr(Y_i > x - m) \sim \sum_{i=1}^{m_0} \Pr(Y_i > x) = \sum_{i=1}^{m_0} \overline{H_i}(x) \tag{10}$$

Using the theory of negatively dependent, we have

$$I_2(x, m) = \Pr(\sum_{i=1}^{m_0} Y_i > x, \frac{x}{m_0} < \max_{1 \leq j \leq m_0} Y_j \leq x - m)$$

$$\leq \sum_{j=1}^{m_0} \Pr(\sum_{i=1}^{m_0} Y_i - Y_j > m, \frac{x}{m_0} < Y_j)$$

$$\leq \sum_{j=1}^{m_0} \sum_{1\leq i\neq j\leq m_0} \Pr(X_i > \frac{m}{m_0-1}, \frac{x}{m_0} < Y_j)$$

$$\leq m_0(m_0-1)\overline{F}(\frac{m}{m_0-1}),\overline{H}(\frac{x}{m_0})_.$$

Therefore,

$$\lim_{m\to\infty}\limsup_{x\to\infty}\frac{I_2(x,m)}{\sum_{i=1}^{m_0}\overline{H_i}(x)}=0 \qquad (11)$$

Combining equations (10) and (11), it is obtained

$$\Pr(\sum_{i=1}^{m_0} X_i e^{-\delta\sigma_i} > x) \leq \sum_{i=1}^{m_0}\Pr(X_i e^{-\delta\sigma_i} > x) \qquad (12)$$

According to (10) and (11), the conclusion is proved.

Lemma 3[11]. If $F \in \mathcal{S}$, $\forall \varepsilon > 0$, there exists a constant coefficient $A(\varepsilon)$ such that

$$\frac{\overline{F^{n*}}(x)}{\overline{F}(x)} \leq A(\varepsilon)(1+\varepsilon)^n_, \quad x \geq 0, n \geq 2_.$$

In the investigation of $F \in \mathcal{L} \cap \mathcal{D}$, Weng et.al. [12] have referred to the following conclusions in Lemma 4.

Lemma 4[11].Ifnon-negativerandom variable series $\{X_i, i \geq 1\}$ follow distribution $F_i \in \mathcal{L} \cap \mathcal{D}$, $i = 1,2,\cdots$, and satisfy equation (5):

$$\lim_{x\to\infty}\frac{\Pr(X_i > x, X_j > x)}{\Pr(X_i > x)}=0_, i\geq 1, j\geq 1,$$

it comes

$$F_1 * F_2 \cdots * F_n(x) \in \mathcal{L}\cap\mathcal{D}$$

and

$$\Pr(\sum_{i=1}^n X_i > x) \sim \sum_{i=1}^n \Pr(X_i > x) \qquad (13)$$

We extend Lemma 4 to subexponential distribution case below.

Corollary 2 We change condition $F \in \mathcal{L}\cap\mathcal{D}$ in lemma 4 to condition $F \in \mathcal{S}$, and lemma 4 is also true.

Proof.When $m_0 = 1$, it is obvious (15)holds. Assuming that $m_0 \geq 2$,clearly,

$$\Pr(\sum_{i=1}^{m_0} X_i e^{-\delta\sigma_i} > x) \geq \Pr(\bigcup_{i=1}^{m_0}(X_i e^{-\delta\sigma_i} > x))$$

$$\geq \sum_{i=1}^{m_0}\Pr(X_i e^{-\delta\sigma_i} > x) - \sum_{1\leq j\neq l\leq m_0}\Pr(X_j e^{-\delta\sigma_j} > x, X_l e^{-\delta\sigma_l} > x)_.$$

When $1 \leq j \neq l \leq m_0$,According to (6), we can easily see that
$$\Pr(X_j e^{-\delta\sigma_j} > x, X_l e^{-\delta\sigma_l} > x) = o(\overline{F}(x))_.$$

Hence,

$$\sum_{1\leq j\neq l\leq m_0}\Pr(X_j e^{-\delta\sigma_j} > x, X_l e^{-\delta\sigma_l} > x) = o(\overline{F}(x))_.$$

Thus,

$$\Pr(\sum_{i=1}^{m_0} X_i e^{-\delta\sigma_i} > x) \geq \sum_{i=1}^{m_0}\Pr(X_i e^{-\delta\sigma_i} > x)_. \qquad (14)$$

For any constant m,

$$\Pr(\sum_{i=1}^{m_0} X_i e^{-\delta\sigma_i} > x) \le$$

$$\Pr(\bigcup_{1\le i\le m_0} X_i e^{-\delta\sigma_i}{}_i > x - m) + \Pr(\sum_{i=1}^{m_0} X_i e^{-\delta\sigma_i} > x, \bigcap_{1\le i\le m_0} X_i e^{-\delta\sigma_i}{}_i \le x - m)$$

$$= I_1(x,m) + I_2(x,m).$$

By Lemma 1, we obtain

$$I_1(x,m) \le \sum_{i=1}^{m_0} \Pr(X_i e^{-\delta\sigma_i}{}_i > x - m) \sim \sum_{i=1}^{m_0} \Pr(X_i e^{-\delta\sigma_i} > x) \tag{15}$$

and

$$I_2(x,m) = \Pr(\sum_{i=1}^{m_0} X_i e^{-\delta\sigma_i} > x, \frac{x}{m_0} < \max_{1\le j\le m_0} X_j e^{-\delta\sigma_j} \le x - m)$$

$$\le \sum_{j=1}^{m_0} \Pr(\sum_{i=1}^{m_0} X_i e^{-\delta\sigma_i}{}_i - X_j e^{-\delta\sigma_j} > m, X_j e^{-\delta\sigma_j} > \frac{x}{m_0})$$

$$\le \sum_{j=1}^{m_0} \sum_{1\le i\ne j\le m_0} \Pr(X_i > \frac{m}{m_0 - 1}, X_j > \frac{x e^{\delta\sigma_j}}{m_0})$$

$$= o(\overline{F}(x)) . \tag{16}$$

Combining (15) and (16), we can get

$$\Pr(\sum_{i=1}^{m_0} X_i e^{-\delta\sigma_i} > x) \le \sum_{i=1}^{m_0} \Pr(X_i e^{-\delta\sigma_i} > x) . \tag{17}$$

According to (14) and (17), the conclusion is proved.

3. Main results with its Proof

Our main results are Theorem 1 and Theorem 2 below.

Theorem 1.With a negatively dependent constant interest force renewal risk model, claims $\{X_i, i \ge 1\}$ are pairwise negatively dependent, where the common distribution function $F \in \mathcal{S}$. The ruin probability of the finite time T satisfies (4), and the ruin probability of the finite time for the negatively dependent claims structure is not sensitive.

Theorem 2.With a constant interest force in the renewal risk model, claims $\{X_i, i \ge 1\}$ is the random variable series which satisfies(5). Its common distribution function $F \in \mathcal{S}$, then the ruin probability of the finite time T satisfies

$$\psi(u,\ T) \sim \int_0^T \overline{F}(u e^{\delta t}) dm(t) \tag{18}$$

Proof of Theorem 1.
Since

$$e^{-\delta T} U_\delta(t) = u + c\int_0^T e^{-\delta y} dy - \sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i}$$

$$= u + \frac{1 - e^{-\delta T}}{\delta} \cdot c - \sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i},$$

we have

$$u - \sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \le e^{-\delta T} U_\delta(t) \le u + \frac{c}{\delta} - \sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i}.$$

Hence,

$$\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \ge u + \frac{c}{\delta}) \le \psi(u,\ T) \le \Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \ge u).$$

If we want to prove (4), we need to prove

$$\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \ge u + \frac{c}{\delta}) \sim \int_0^T \overline{F}(u e^{\delta t}) dm(t) \sim \Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \ge u) \tag{19}$$

Since

$$\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u) = \sum_{k=N_0+1}^{\infty} \Pr(\sum_{i=1}^{k} X_i e^{-\delta\sigma_i} \geq u, N(T)=k)$$

$$+ \sum_{k=1}^{N_0} \Pr(\sum_{i=1}^{k} X_i e^{-\delta\sigma_i} \geq u, N(T)=k) = I_1(x) + I_2(x)$$

It follows from Corollary 1 and Lemma 3, obtained

$$I_1(x) = \sum_{k=N_0+1}^{\infty} \sum_{i=1}^{k} \Pr(X_i e^{-\delta\sigma_i} \geq u, \sigma_k = \sum_{j=1}^{k} \theta_j \leq T)$$

$$\leq A(\varepsilon) \int_0^T \overline{F}(ue^{\delta t}) \sum_{k=N_0+1}^{\infty} (1+\varepsilon)^k \Pr(N(T-t)=k) dF_\theta(t)$$

$$\leq A(\varepsilon) E\left[(1+\varepsilon)^{N(T)} 1_{(N(T)\geq N_0)}\right] \int_0^T \overline{F}(ue^{\delta t}) dF_\theta(t)$$

According to Chebyshev inequality, we have

$$E\left[(1+\varepsilon)^{N(T)} 1_{(N(T)\geq N_0)}\right] \leq \sum_{k=N_0}^{\infty} \left((1+\varepsilon) E e^{-\theta}\right)^k e^T$$.

Thus, there exists ε such that $(1+\varepsilon) E e^{-\theta} < 1$, for any $\eta_0 > 0$, and there exists N_0 such that $A(\varepsilon) E\left[(1+\varepsilon)^{N(T)} 1_{(N(T)\geq N_0)}\right] < \eta_0$. Then

$$I_1(x) \leq \eta_0 \int_0^T \overline{F}(ue^{\delta t}) dm(t)$$.   (20)

So, $I_2(x) \sim \sum_{k=1}^{N_0} \sum_{i=1}^{k} \Pr(X_i e^{-\delta\sigma_i} \geq u, N(T)=k) \leq \sum_{k=1}^{\infty} \sum_{i=1}^{k} \Pr(X_i e^{-\delta\sigma_i} \geq u, N(T)=k)$

$$= \int_0^T \overline{F}(ue^{\delta t}) dm(t)$$.   (21)

According to (20) and (21), we can get

$$\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u) \leq (1+\eta_0) \int_0^T \overline{F}(ue^{\delta t}) dm(t)$$ (22)

and

$$\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u) = \sum_{k=1}^{\infty} \Pr(\sum_{i=1}^{k} X_i e^{-\delta\sigma_i} \geq u, N(T)=k)$$

$$\geq \sum_{k=1}^{N_0} \sum_{i=1}^{k} \Pr(X_i e^{-\delta\sigma_i} \geq u, N(T)=k)$$ (23)

$$\geq (1-\eta_0) \int_0^T \overline{F}(ue^{\delta t}) dm(t)$$

Since η_0 is arbitrary, it comes $\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u) \sim \int_0^T \overline{F}(ue^{\delta t}) dm(t)$. It follows from Lemma 1 that $\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u + \frac{c}{\delta}) \sim \Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u)$. Thus, (4) is proved.

Proof of Theorem 2.

Since

$$e^{-\delta T} U_\delta(t) = u + c \int_0^T e^{-\delta y} dy - \sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} = u + \frac{1-e^{-\delta T}}{\delta} \cdot c - \sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i},$$

we have

$$u - \sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \leq e^{-\delta T} U_\delta(t) \leq u + \frac{c}{\delta} - \sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i}$$.

Then,

$$\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u + \frac{c}{\delta}) \leq \psi(u, T) \leq \Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u)$$.

If we want to prove (18), it is suffice to prove

$$\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u + \frac{c}{\delta}) \sim \int_0^T \overline{F}(ue^{\delta t}) dm(t)$$

$$\sim \Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u) \tag{24}$$

Since

$$\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u) = \sum_{k=N_0+1}^{\infty} \Pr(\sum_{i=1}^{k} X_i e^{-\delta\sigma_i} \geq u, N(T) = k)$$

$$+ \sum_{k=1}^{N_0} \Pr(\sum_{i=1}^{k} X_i e^{-\delta\sigma_i} \geq u, N(T) = k) = I_1(x) + I_2(x) \quad ,$$

because ofCorollary 2 and Lemmas 3, that come

$$I_1(x) = \sum_{k=N_0+1}^{\infty} \sum_{i=1}^{k} \Pr(X_i e^{-\delta\sigma_i} \geq u, \sigma_k = \sum_{j=1}^{k} \theta_j \leq T)$$

$$\leq A(\varepsilon) \int_0^T \overline{F}(ue^{\delta t}) \sum_{k=N_0+1}^{\infty} (1+\varepsilon)^k \Pr(N(T-t) = k) dF_\theta(t)$$

$$\leq A(\varepsilon) E\left[(1+\varepsilon)^{N(T)} 1_{(N(T) \geq N_0)}\right] \int_0^T \overline{F}(ue^{\delta t}) dF_\theta(t) \quad .$$

According to Chebyshev inequality, for any $\eta_0 > 0$,there exists N_0 such that

$$I_1(x) \leq \eta_0 \int_0^T \overline{F}(ue^{\delta t}) dm(t) \tag{25}$$

and

$$I_2(x) \sim \sum_{k=1}^{N_0} \sum_{i=1}^{k} \Pr(X_i e^{-\delta\sigma_i} \geq u, N(T) = k)$$

$$\leq \sum_{k=1}^{\infty} \sum_{i=1}^{k} \Pr(X_i e^{-\delta\sigma_i} \geq u, N(T) = k) \cdot \tag{26}$$

$$= \int_0^T \overline{F}(ue^{\delta t}) dm(t)$$

Combining (25) with (26),one can easily see that

$$\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u) \leq (1+\eta_0) \int_0^T \overline{F}(ue^{\delta t}) dm(t) \tag{27}$$

and

$$\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u) = \sum_{k=1}^{\infty} \Pr(\sum_{i=1}^{k} X_i e^{-\delta\sigma_i} \geq u, N(T) = k)$$

$$\geq \sum_{k=1}^{N_0} \sum_{i=1}^{k} \Pr(X_i e^{-\delta\sigma_i} \geq u, N(T) = k) \cdot \tag{28}$$

$$\geq (1-\eta_0) \int_0^T \overline{F}(ue^{\delta t}) dm(t)$$

As η_0 is arbitrary, we have

$$\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u) \sim \int_0^T \overline{F}(ue^{\delta t}) dm(t) \quad .$$

By Lemma 1,

$$\Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u + \frac{c}{\delta}) \sim \Pr(\sum_{i=1}^{N(T)} X_i e^{-\delta\sigma_i} \geq u) \quad ,$$

then (18) is proved.

Acknowledgements

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