

Research on dynamic mean-variance portfolio selection

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Abstract. The covariance matrix of portfolio returns constructed by static mean-variance model will change with time, so the optimal portfolio with fixed weights may not be optimal. Based on the static mean-variance model, this paper introduces two new parameters of the length of dynamic historical period and holding period to construct a dynamic mean-variance model. Numerical analysis shows that the proposed dynamic portfolio strategy can achieve good returns.

Keywords: Quantitative investment; Mean-variance; Dynamic mean-variance.

1. Introduction

Markowitz introduced the mean-variance theory, establishing a theoretical foundation for evaluating the association between asset risk and return, as well as methodology for risk diversification [1]. Later, Merton studied the continuous time portfolio model and gave the analytic solution of the optimal portfolio with hyperbolic absolute risk aversion utility function[2]. Harrison et al. studied the problems of portfolio by martingale method. The dynamic programming technology is used to study the dynamic portfolio with constraints of state variables or control variables[3,4]. Pliska transformed continuous time portfolio model into an equivalent optimal programming problem by martingale method. The problem can be easily solved by introducing a special portfolio[5]. Bajerx-Besnaoui and Portrait studied the dynamic portfolio using martingale method within the framework of mean-variance theory[6]. Richardson tested the mean-variance model under both static and dynamic conditions, verifying that a multi-period dynamic portfolio strategy is superior to a static portfolio strategy[7]. Li and Ng creatively transformed multi-stage mean-variance model into an optimization problem using dynamic programming technology and obtained the analytical formula of optimal portfolio strategy[8].

In this paper, a dynamic mean-variance portfolio model is constructed by introducing the length of reference historical time period and holding period, and is verified by taking blue chip stocks as an example. The innovation and research contribution of this paper are as follows: (1) In the dynamic mean-variance portfolio model, two parameters of reference historical time period and holding period are introduced to obtain the path to realize the income of the mean-variance portfolio in each stage. (2) Different from Li and Ng's research, this paper considers the investment weight of the portfolio and the frequency with which the investment weight is adjusted. The adjustment frequency is reflected by holding period, and the model after the introduction of this parameter is more in line with the actual investment. (3) According to the actual data of China's securities market, starting from the effective frontier of dynamic portfolio, this paper analyzes the efficiency of dynamic portfolio under different parameters and selects the most reasonable parameters. The results show that the dynamic model and traversing method proposed in this paper are very effective and feasible.

2. Model

2.1 Dynamic portfolio model

2.1.1 Return, variance and covariance of each period in a multi-period dynamic portfolio mean-variance model

The length of historical time period is X , and the length of holding period is Δt , both of which are used to construct the dynamic mean-variance model. At the beginning of the investment, if there are n trading days of historical data, then the investors at the beginning of the investment time $t = n$, and the range of the historical time period is $[n - X + 1, n]$. Over time, the window of this

historical time period gradually moves forward in the investment process. The expected return of each asset i in the portfolio at time $t = n$ is:

$$\bar{r}_i = \sum_{t=1}^X r_{i,n+1-t} / X \tag{1}$$

The risk of each asset i in a portfolio can be represented by the variance of its yield:

$$\sigma_i^2 = \sum_{t=1}^X (r_{i,n+1-t} - \bar{r}_i)^2 / X \tag{2}$$

The variance between any two assets i and j in the portfolio is:

$$\sigma_{ij} = E[(r_{i,t} - \bar{r}_i)(r_{j,t} - \bar{r}_j)] = \sum_{t=1}^X (r_{i,n+1-t} - \bar{r}_i)(r_{j,n+1-t} - \bar{r}_j) / X \tag{3}$$

Based on formulas (1) to (3), when $X = n$, the above multi-period portfolio model is consistent with the single period portfolio model.

2.1.2 Consider portfolio weight and adjustment frequency

If the total investment period is S and the whole investment period is E , then $E = S\Delta t + \varepsilon$ will be satisfied from the initial time $t = n$ to the end of the investment, where ε is the remaining time at the end of the last investment cycle. In other words, during the holding period of interval Δt , the weight of the stocks in the portfolio can be adjusted S times in the stock pool.

During investors' holding the portfolio, considering the weight and adjustment frequency of the dynamic asset portfolio, m is the number of times of investment weight adjustment at the current time. During the m adjustment period, the expected return and risk of each asset measured by the average and variance of historical return rate are as follows:

$$\bar{r}_{i,n+m\Delta t} = \sum_{t=1}^X r_{i,n+m\Delta t-t+1} / X \tag{4}$$

$$\sigma_{i,n+m\Delta t}^2 = \sum_{t=1}^X (r_{i,n+m\Delta t-t+1} - \bar{r}_{i,n+m\Delta t})^2 / X \tag{5}$$

At this point, the expected return and risk of the portfolio are as follows:

$$\bar{r}_{P,n+m\Delta t} = \sum_{i=1}^N w_{i,n+m\Delta t} \bar{r}_{i,n+m\Delta t} \tag{6}$$

$$\sigma_{P,n+m\Delta t}^2 = \sum_{i=1}^N w_{i,n+m\Delta t}^2 \sigma_{i,n+m\Delta t}^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N w_{i,n+m\Delta t} w_{j,n+m\Delta t} \sigma_{ij,n+m\Delta t} \tag{7}$$

The variance $\sigma_{ij,n+m\Delta t}$ between asset i and asset j in formula (7) can be calculated by the following formula (8):

$$\sigma_{ij,n+m\Delta t} = \sum_{t=1}^X (r_{i,n+m\Delta t-t+1} - \bar{r}_{i,n+m\Delta t})(r_{j,n+m\Delta t-t+1} - \bar{r}_{j,n+m\Delta t}) / X \tag{8}$$

In summary, taking into account the portfolio weight, the frequency of investment strategy adjustment and the risk aversion coefficient of investors, the multi-period dynamic portfolio model is as follows:

$$\begin{aligned} \max U_{n+m\Delta t} &= \bar{r}_{P,n+m\Delta t} - 1/2A\sigma_{P,n+m\Delta t}^2 \\ \text{s.t.} &\begin{cases} \sum_{i=1}^N w_{i,n+m\Delta t} = 1 \\ w_{i,n+m\Delta t} \geq 0, \quad i = 1, 2, \dots, N; \end{cases} \end{aligned} \tag{9}$$

In formula (9), according to the risk aversion level A , the optimal investment weight is adjusted dynamically by adopting the appropriate length of historical time period X and holding period Δt , so as to ensure the best investment return in the investment process.

3. Simulation, sensitivity analysis and validation of dynamic portfolio model

3.1 Simulation of dynamic portfolio model

In order to simulate, calibrate and verify the effectiveness of the dynamic portfolio model, ten blue chip stocks in China's stock market from 1 January 2,018 to 22 January 2,020 were used as sample data., A total of 243 trading days from 1 January 2,018 to 1 January 2,019 were used as the historical reference period for the selection of blue chip stocks and the expected return on assets. A total of 259 trading days from 2 January 2,019 to 22 January 2,020 were used as the investment period to construct and dynamically adjust our asset portfolio and set the investor risk aversion factor $A = 4$. In calculating the effective front, 100 points were used to describe the curve of the effective front.

The distribution of all the yields in the portfolio $(X, \Delta t)$ of the length of all the reference historical time periods X and that of holding period Δt is shown in Fig. 1. As shown in Fig. 1, in the two-dimensional plane region composed of the length of reference historical time period X and the that of holding period Δt , the return of portfolio varies greatly among sub regions.

Specifically, (1) The yield in the left, lower and lower left corners of Fig. 1 is small, which is about 20%, and the area of the same yield in these areas is small, which indicates that when the length of reference historical time period X is short, that of holding period Δt is short, or the length of two periods is short at the same time, the yield calculated by these parameters is low and unstable.

(2) In Fig. 1, the yield in the middle and upper right corners is higher than that in the above areas, and the areas of the same yield area increase. In this region, the yield is stable at around 70% in many places, and even reaches 100% in some places, which indicates that the yield increases significantly, and then stabilizes with the gradual increase of the length of reference historical time period X and that of holding period Δt .

(3) Fig. 1 presents the difference of yield in different regions, which indicates that the setting of $(X, \Delta t)$ has a great impact on the yield of portfolio in the dynamic mean-variance model.

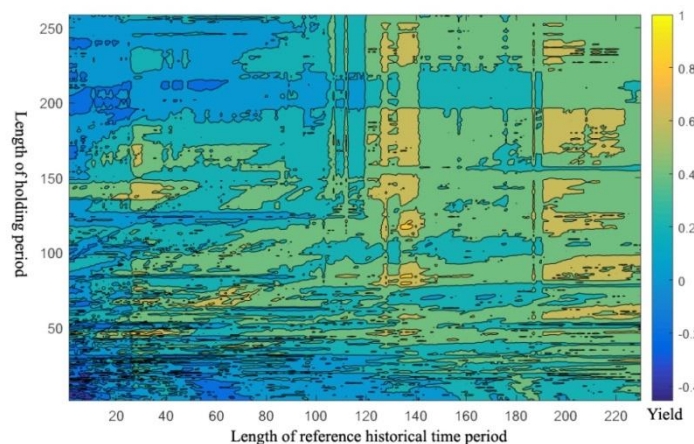


Fig. 1. Traversal of portfolio returns with reference to historical time periods and holding periods

3.2 Sensitivity analysis of dynamic investment model

According to the previous analysis, when the length of holding period Δt is short, the yield is in a lower range, suggesting that the frequently adjusted position weight will not lead to an increase in transaction cost instead of income. This section examines how the weight of the optimal portfolio

changes when the holding period is short. In particular, we take $X = 100$, $\Delta t = 1$ and the initial capital is 100,000. The short holding cycle allows more holding data points to observe the changes in weight and capital.

Fig. 2 shows the change in the weight of the optimal portfolio with a shorter holding period. As shown in Fig. 2, in the calculation of parameter iteration, the weight of some stocks in the entire asset portfolio is always low, which indicates that these stocks have not been held in the whole investment process. In Fig. 2, the stocks with higher weight in the entire asset portfolio are the first, second, third and 6 stocks.

This section also examines the changes in the amount of funds in the portfolio with a short holding period. Fig. 3 shows the curve of change of funds in the holding period with this parameter over time, which is consistent with the final yield shown in Fig. 1.

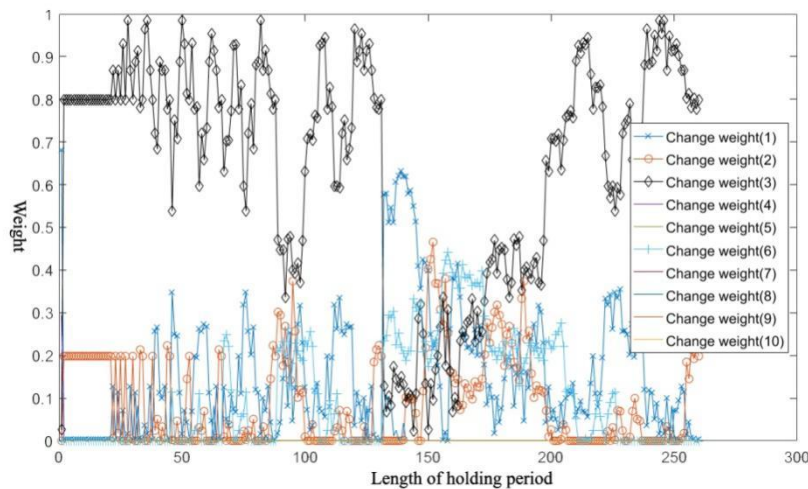


Fig. 2 Changes of weight in short holding period

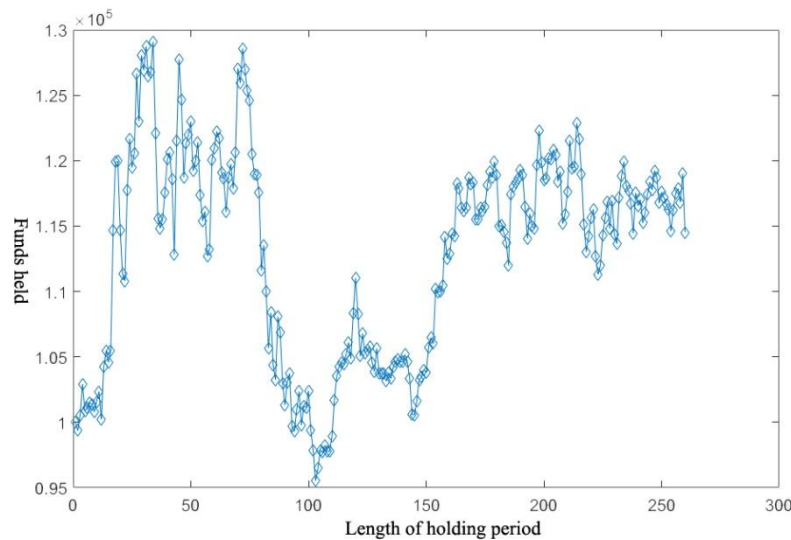


Fig. 3 Curve of funds in short holding period

3.3 Validation of dynamic investment model

In order to verify the validity of the dynamic investment model constructed in this paper, we select a high-return region from Fig. 1. In particular, the parameter range of X is 130-140 and that of Δt is 160-200. The reason for selecting this region is that the return rate of the portfolio in this region is relatively high, and the region of the same return rate is larger than that in the lower left corner.

Similarly, it is assumed that the investment will start on January 2, 2019 and end on January 22, 2020. In this period, when the parameters are $X = 136$, $\Delta t = 168$, then the return of portfolio is 82.9%, and the annualized return is 78.6%. During the same period, the CSI 300 index rose from 2969.54 to 4131.93, with an index yield of 39.1% and an annualized yield of 37.1%. Comparing the return of dynamic portfolio with that of CSI 300 index in the same period, it is not difficult to find that the dynamic portfolio strategy achieves better returns and excess returns by optimizing stocks and selecting appropriate cycle parameters.

4. Summary

The classic mean-variance method proposed by Markowitz faces a severe challenge that it is a static single-period model and cannot be applied to multi-stage or continuous time in the real world. This paper constructs a dynamic mean-variance portfolio model. The innovation and value of dynamic portfolio model lies in its more general dynamic meaning, which is mainly reflected in the following aspects: Firstly, the historical reference time period used to calculate the mean and variance is continuously rolling; secondly, investment weight is constantly changing throughout the investment period. Therefore, the method of establishing parameters in the construction of the model is of great theoretical significance for expanding and improving mean-variance model.

Acknowledgement

Supported by Research Fund of Sanda University, China (2022BSZX06)

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