# Pricing Decision and Cancellation Strategy Selection of 'Hotel+OTA' Based on Customer Cancellation Behavior 

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#### Abstract

This paper is based on customer cancellation behavior and combines the Hotelling model framework to construct the demand for hotel direct sales channels and online travel agency (OTA) online channels. Then, the Stackelberg game model is used to analyze the optimal pricing problem between hotels and OTA with competitive relationships, and the optimal cancellation strategy for hotel direct channel is analyzed. We demonstrate that the hotel's most profitable cancellation strategy depend on the loss cost of customer cancellation to the hotel, the cost of channel transfer for customers, and the OTA's monopoly market segment. When the cost of loss to the hotel due to customer cancellation is relatively small, the hotel chooses a strategy where customers can cancel their reservations (i.e. Y strategy), and the pricing of hotel rooms increases as the cost of cancellation loss increases; otherwise, the hotel chooses a strategy where the customer cannot cancel the reservation (i.e. N strategy), and the pricing of hotel rooms decreases as the cost of cancellation loss increases. When the cost of channel transfer for customers is low, the hotel chooses N strategy; otherwise, the hotel chooses Y strategy. When the OTA's monopoly market segment (i.e., the customers attracted solely to the OTA channel) is large, the hotel chooses N strategy; otherwise, the hotel chooses Y strategy. These findings can assist hotel managers in improving the design of their pricing and cancellation strategy.


Keywords:cancellation behavior, OTA, cancellation strategy, Stackelberg game.

## 1. Introduction

With the change in the social environment, the frequency of people's travel gradually recovers, and the tourism industry has also picked up rapidly, followed by the gradual revival of all walks of life in society, and the hotel industry is also in its ranks. Now everyone's travel demand more and more strong, the demand for hotel rooms is becoming more and more strong, but consumer consumption may be affected by many factors to cancel the order, such as consumer travel termination or change, or other more valuable hotel reservation, or the hotel room reservation and insufficient customer schedule changes.

The concept of unsubscription first appeared in the service industry, such as hotel pre-sale, ticket pre-sale, etc. ${ }^{[1]}$, that is, the behavior of consumers canceling the service before using the service, and getting a certain price compensation. Due to consumers' varying valuations of products and their perception of them, merchants will use refund and return services to reduce the uncertainty of consumers' valuations of products and promote their own profits not to be affected ${ }^{[2,3]}$. The strategy of refunding and returning goods adopted by merchants may put them at risk ${ }^{[4]}$, but overall, it is beneficial for merchants ${ }^{[5-8]}$. Of course, in different situations, merchants may adopt different unsubscribe strategies, such as developing strategies based on product price differences ${ }^{[9]}$ or adopting joint strategies ${ }^{[10]}$. From a consumer perspective, whether to unsubscribe from a product depends on whether there are more valuable products or services available. Assuming that the probability of consumers' unsubscription is fixed, then a reasonable income model can be designed to formulate the optimal pricing ${ }^{[11]}$. Establishing a reasonable refund ratio has a significant impact on merchants' own profits ${ }^{[12,13]}$, and at the same time, consumer preferences can also affect the formulation of merchants' unsubscribe strategies ${ }^{[14]}$. From the perspective of the supply chain, the decisions of its participating entities are interdependent ${ }^{[15]}$ and are influenced by channel search
costs, consumer preferences ${ }^{[16]}$, supply channel efficiency, and risks ${ }^{[17]}$. As for the supply chain of hotel, the rapid development of Internet technology has led to the expansion of its supply channels. Due to the convenience of online channels, the probability of consumers unsubscribing will also be greater ${ }^{[18]}$, and the cooperation mode of online and offline entities will also affect the Pricing of channels ${ }^{[19]}$. Of course, the main revenue of a hotel comes from consumers' check-in, so consumers' unsubscription will seriously affect the overall revenue of the hotel ${ }^{[20]}$, and the hotel's room pricing will also affect the demand for rooms, thereby affecting the hotel's revenue ${ }^{[21]}$.

Currently, hotels can sell rooms through OTA and their own direct sales channels, but they have different cancellation strategies for orders. Each cancellation strategy has its own advantages and disadvantages. At the same time, the different cancellation strategies of OTA and hotels in dualchannel sales can affect consumers' purchasing intentions. Therefore, the research question raised in this paper is the research on the cancellation strategy selection and Pricing of hotels in the context of the dual channel competition between hotels and OTAs. The solution of this research problem can help the hotel to make better strategic planning, provide a better choice of Pricing and channel strategy for the hotel, and thus bring higher economic benefits.

## 2. The Model

### 2.1 Problem Description

This paper studies the supply chain consisting of a hotel and an OTA, the hotel can sell rooms through its direct channel or the OTA's indirect channel, so there is both cooperation and competition between the hotel and the OTA. And the hotel cooperates with it through the wholesale model. Under the wholesale model, the OTA promises to sell a certain number of rooms every month and prepays the hotel at an agreed price; the OTA then sets the retail price offered to consumers. Thus, the decision-making process of the hotel and the OTA is a two-stage Stackelberg game; the hotel acts as a Stackelberg leader and the OTA acts as a follower ${ }^{[22]}$. Thus, the hotel determines the optimal direct channel price $\mathrm{P}_{\mathrm{h}}$ and wholesale price $\mathrm{P}_{\mathrm{w}}$ to maximize profits, and then the OTA determines its optimal retail price $P_{o}$ based on the hotel's wholesale price. subscript $h(o)$ denotes a variable pertaining to the hotel (OTA).

In addition, during the process of selling rooms in the hotel and the OTA, consumers may cancel their rooms for various reasons, both the hotel and the OTA have two cancellation strategies to choose from (i.e. cancelable strategy and non-cancelable strategy). Therefore, the hotel needs to make its optimal cancellation strategy while considering the OTA's cancellation strategy. The superscript $Y(N)$ indicates that the hotel or OTA adopts a strategy that consumers can cancel their reservations (non-cancellable strategy).

### 2.2 Basic Assumptions

Some assumptions are required to simplify the model.

1. Without loss of generality, the price of the hotel direct channel and the OTA sales price are both higher than the wholesale price, that is, $P_{h}>P_{w}>0, P_{o}>P_{w}>0$.
2. Assuming the total market demand scale is 1 , then the r proportion of customers have access to both the hotel direct channel and the OTA channel and will compare the efficiency of purchasing from both channels to make decisions, while $1-r$ represents how much market scale can be increased by increasing online travel agency channels.
3. As customers need to reserve rooms, when both the hotel and the OTA allow customers to cancel the reservation, customers' cancellation behavior will bring unit loss $C$ to them. When they don't allow customers to cancel the reservation, customers will have a perceived cost $\beta * P(0<$ $\beta<1$ ).
4. Assuming that the customer's perceived value of the guest room is $V$, according to the consumer utility theory, the customer utility $U=V-P_{h}-t(1-d)$ for the hotel direct channel to
book the guest room, and the customer utility $U=V-P_{o}-t d$ for the OTA channel, where $0<$ $V<1, d$ is evenly distributed in $[0,1], t$ is the unit cost of channel transfer for customers, and $d^{*}\left(d^{*}=\left(P_{h}-P_{o}+t\right) / 2 t\right)$ is the indifference point between the hotel direct channel and OTA channel..
5.Consumers are evenly distributed within the spatial range of [0,1], with hotel direct cha nnels at end 1 and OTA channels at end 0 . The demand for hotel direct channel is $D_{h}$, wh ile the demand for OTA is $D_{o}, D_{h}=r\left(P_{o}-P_{h}+t\right) /(2 t), D_{o}=r\left(P_{h}-P_{o}+t\right) /(2 t)+(1-r)(1-$ $P_{o}$ )

## 3. Model Analysis

Based on the above assumptions, there are four combinations of decision models: YY, YN, NY, and NN. The following will discuss the pricing and profit of the hotel and the OTA under the four models respectively.

### 3.1 Y-Y Model

When both channels allow customers to unsubscribe, there is no perceived cost $\beta * P_{h}$ and $\beta * P_{o}$. According to the above assumptions, getting the profit functions of the hotel and the OTA:

$$
\left\{\begin{array}{c}
\pi_{h}^{Y Y}=\left(P_{h}^{Y Y}-C\right) * r\left(P_{o}^{Y Y}-P_{h}^{Y Y}+t\right) /(2 t)+P_{w} *\left[r\left(P_{h}^{Y Y}-P_{o}^{Y Y}+t\right) /(2 t)+(1-r)\left(1-P_{o}^{Y Y}\right)\right] \\
\pi_{o}^{Y Y}=\left(P_{o}^{Y Y}-P_{w}^{Y Y}-C\right) *\left[r\left(P_{h}^{Y Y}-P_{o}^{Y Y}+t\right) /(2 t)+(1-r)\left(1-P_{o}^{Y Y}\right)\right]
\end{array}\right.
$$

Since the hotel is the dominant decision-making and the OTA is the follower, according to the reverse solution method of the Stackelberg game model, then the optimal price of the hotel and OTA are as expressed:
the optimal price of the OTA channel:

$$
P_{o}^{* Y Y}=t(1-r)[2 C(1-r)-r+6]+2 r[1+C(1-r)] / 4(1-r)[2 t(1-r)+r]
$$

the optimal price of the hotel direct channel: $P_{h}^{* Y Y}=1 / 2(1-r)+C+t / 2$;
the optimal price of the wholesale price: $P_{w}^{* Y Y}=1 / 2(1-r)-C / 2$.
The $P_{h}^{* Y Y}, P_{w}^{* Y Y}, P_{o}^{* Y Y}$ was added into the profit function: $\pi_{h}^{Y Y}$ and $\pi_{o}^{Y Y}$, get the profit value of the hotel and the OTA platform:

$$
\left\{\begin{array}{c}
\pi_{h}^{* Y Y}=\left\{4 r[1-C(1-r)]^{2}+4 r t^{2}(1-r)^{2}+t(1-r)[2-2 C(1-r)+r]^{2}\right\} / 16[2 t(1-r)+r] \\
\pi_{o}^{* Y Y}=\left\{t[2-2 C(1-r)-r]^{2}\right\} / 32[2 t(1-r)+r]
\end{array}\right.
$$

Lemma 1. In the case of $Y-Y$ model, if other parameters remain unchanged, the price of the hotel direct channel $P_{h}^{* Y Y}$ increases with the increase of $C$ and $t$, otherwise, it decreases. The price of hotel wholesale rooms to the OTA $P_{w}^{* Y Y}$ is a decreasing function of $C$. When $0<C<1-r / 2(1-r)$, the OTA channel price $P_{o}^{* Y Y}$ is an increasing function of $t$, when $1-r / 2(1-r)<C<1, P_{o}^{* Y Y}$ is a decreasing function of $t$.
All proofs are given in the Appendix.
Lemma 2. In the case of $Y-Y$ model, with other parameters remain unchanged, when $0<C<$ $\left[t\left(2-r^{2}\right)+r(2-t)\right] /\{2 t(1-2 r)+2 r[1-r(1-t)]\}$, the profit of hotel direct channel $\pi_{h}^{* Y Y}$ is a decreasing function of $C$, when $\left[t\left(2-r^{2}\right)+r(2-t)\right] /\{2 t(1-2 r)+2 r[1-r(1-t)]\}<C<1$, the $\pi_{h}^{* Y Y}$ is a increasing function of $C$. When $0<C<(2-r) / 2(1-r)$, the profit of OTA $\pi_{o}^{* Y Y}$ are the decreasing function of $C$; when $(2-r) / 2(1-r)<C<1$, the $\pi_{o}^{* Y Y}$ are the increasing function of $C$.

### 3.2 Y-N Model

When the hotel can be canceled and the OTA cannot be canceled, the customer has the perceived cost of subscription $\beta * P_{o}$. At this time, the customer utility of selecting the hotel direct channel is $U_{h}=V-P_{h}-t(1-d)$, and the customer utility of selecting OTA is $U_{o}=V-P_{o}-t d-\beta P_{o}$. Therefore, the indifference point is $d^{*}=\left[P_{h}-(1+\beta) P_{o}+t\right] / 2 t$, according to assumptions, getting the profit functions of the hotel and the OTA are respectively:

$$
\left\{\begin{array}{c}
\pi_{h}^{Y N}=\left(P_{h}^{Y N}-C\right) * r\left[(1+\beta) P_{o}^{Y N}-P_{h}^{Y N}+t\right] / 2 t+P_{w}^{Y N} *\left\{r\left[P_{h}^{Y N}-(1+\beta) P_{o}^{Y N}+t\right] / 2 t+(1-r)\left[1-(1+\beta) P_{o}^{Y N}\right]\right\} \\
\pi_{o}^{Y N}=\left(P_{o}^{Y N}-P_{w}^{Y N}\right)\left\{r\left[P_{h}^{Y N}-(1+\beta) P_{o}^{Y N}+t\right] / 2 t+(1-r)\left[1-(1+\beta) P_{o}^{Y N}\right]\right\}
\end{array}\right.
$$

Similar to Section 3.1, we can get the optimal price of the hotel and OTA as expressed: the optimal price of the OTA channel:

$$
P_{o}^{* Y N}=\left\{24 t(t-r)+2 \operatorname{tr}(1-r)(C-t)(2-\beta)-\beta r^{2}(C-t)+8 t r\right\} /[2 t(1-r)+r]\left[16 t(1-r)(1+\beta)-\beta^{2} r\right]
$$

the optimal price of the hotel direct channel:

$$
P_{h}^{* Y N}=\left\{C \beta r-8 t^{2}(1-r)(1+\beta)-t[8+(6+r) \beta+8 C(1-r)(1+\beta)]\right\} /\left[r \beta^{2}-16 t(1-r)(1+\beta)\right]
$$

the optimal price of the wholesale price:

$$
P_{w}^{* Y N}=\left\{4 t^{2}(1-r)(4+r \beta)-\operatorname{tr}[8+\beta(2+r-4 C(1-r))]+C \beta r^{2}\right\} /[2 t(1-r)+r]\left[r \beta^{2}-16 t(1-r)(1+\beta)\right]
$$

The $P_{h}^{* Y N}, P_{w}^{* Y N}, P_{o}^{* Y N}$ was added into the profit function: $\pi_{h}^{Y N}$ and $\pi_{o}^{Y N}$, get the profit value of the hotel and the OTA:

$$
\left\{\begin{array}{c}
\pi_{h}^{* Y N}=\frac{r C^{2}(1-r)(1+\beta)[4 t(1-r)+r]}{\left[16 t(1-r)(1+\beta)-\beta^{2} r\right][2 t(1-r)+r]}-\frac{r C\left[r \beta+8 t^{2}(1-r)^{2}(1+\beta)+2 t(1-r)(2+3 \beta+r+r \beta)\right]}{\left[16 t(1-r)(1+\beta)-\beta^{2} r\right][2 t(1-r)+r]}+ \\
\frac{t\left\{4 t+t r^{3}(4 t-1)(1+\beta)+2 r\left[2+\beta+3 t \beta+2 t^{2}(1+\beta)\right]+r^{2}\left[\beta-8 t^{2}(1+\beta)-3 t-5 t \beta\right]\right\}}{\left[16 t(1-r)(1+\beta)-\beta^{2} r\right][2 t(1-r)+r]} \\
\pi_{o}^{* Y N}=\frac{2 t(1+\beta)\{r \beta-C r(1-r)(2+\beta)-t(1-r)[4-r(2+\beta)]\}^{2}}{[2 t(1-r)+r]\left[16 t(1-r)(1+\beta)-\beta^{2} r\right]^{2}}
\end{array}\right.
$$

Lemma 3. In the case of $Y-N$ model, when $1>t>r / 32(1-r)$, there is only an equilibrium solution $P_{h}^{* Y N}, P_{w}^{* Y N}, P_{o}^{* Y N}$ make the profit functions $\pi_{h}^{Y N}, \pi_{o}^{Y N}$ have the optimal value.

Corollary 1: When $1>t>r / 32(1-r)$ and $0<\beta<1 / 4$, the optimal price of the hotel direct channel $P_{h}^{* Y N}$ is an increasing function of $C$, when $1>t>r / 32(1-r)$ and $8 t(1-r) / r<\beta<1$, $P_{h}^{* Y N}$ is a decreasing function of $C$.

Corollary 2: With other parameters unchanged, when $1>t>r / 32(1-r)$ and $0<\beta<2 / 17$, the optimal price of the OTA $P_{o}^{* Y N}$ is an increasing function of $C$, when $1>t>r / 32(1-r)$ and $4 t(1-r) / 2 t(1-r)+r<\beta<1, P_{o}^{* Y N}$ is a decreasing function of $C$.

Corollary 3: With other parameters unchanged, when $1>t>r / 32(1-r)$, the optimal wholesale price $P_{w}^{* Y N}$ is a decreasing function of $C$.

### 3.3 N-Y Model

When the hotel cannot be cancelled and the OTA can be cancelled, the customer has the subscription perception cost $\beta * P_{h}$. At this time, the customer utility of the hotel direct channel is $U_{h}=V-(1+\beta) P_{h}-t(1-d)$, and the customer utility of the OTA channel is $U_{o}=V-P_{o}-t d$. Therefore, the indifference point is $d^{*}=\left[(1+\beta) P_{h}-P_{o}+t\right] / 2 t$, according to assumptions, the profit functions of the hotel and the OTA are respectively:

$$
\left\{\begin{array}{c}
\pi_{h}^{N Y}=P_{h}^{N Y} * r\left[P_{o}^{N Y}-(1+\beta) P_{h}^{N Y}+t\right] / 2 t+P_{w} *\left\{r\left[(1+\beta) P_{h}^{N Y}-P_{o}^{N Y}+t\right] / 2 t+(1-r)\left(1-P_{o}^{N Y}\right)\right\} \\
\pi_{o}^{N Y}=\left(P_{o}^{N Y}-P_{w}^{N Y}-C\right)\left\{r\left[(1+\beta) P_{h}^{N Y}-P_{o}^{N Y}+t\right] / 2 t+(1-r)\left(1-P_{o}^{N Y}\right)\right\}
\end{array}\right.
$$

Similar to Section 3.1, we can get the optimal price of the hotel and OTA as expressed: the optimal price of the OTA channel:

$$
P_{o}^{* N Y}=\left\{t(1+\beta)\{8 t(3+C)+r[8-2 C(8 t+\beta-r)]\}+\operatorname{tr}[4 C-t(28-3 \beta-4 r)+r \beta(6 t-1)] /\left[16 t(1-r)(1+\beta)-r \beta^{2}\right][2 t(1-r)+r]\right.
$$

the optimal price of the hotel direct channel:

$$
P_{h}^{* N Y}=\left\{8 t^{2}(1-r)-r C \beta+8 t+t \beta[2-r-2 C(1-r)]\right\} /\left[16 t(1-r)(1+\beta)-r \beta^{2}\right]
$$

the optimal price of the wholesale price:

$$
P_{w}^{* N Y}=\left\{8 \operatorname{tr}(1-C)+16 t^{2} r[2 C(1+\beta)-1]-4 \operatorname{tr} \beta(5 t+1)\right\} /\left[16 t(1-r)(1+\beta)-r \beta^{2}\right][2 t(1-r)+r]
$$

The $P_{h}^{* N Y}, P_{w}^{* N Y}, P_{o}^{* N Y}$ was added into the profit function: $\pi_{h}^{N Y}$ and $\pi_{o}^{N Y}$, we can get the profits of the hotel and the OTA:
$\left\{\begin{array}{c}\pi_{h}^{* N Y}=\frac{C(1-r)(1+\beta)[2 t(1-r)+r]\{C[2 t(1-r)+r]+t(4-r)\}+[t \beta(2-r)-r C(1-\beta)][2 t(1-r)+r]+4 t^{2}\left[t r(1-r)^{2}+1\right]+\operatorname{tr}[4-\operatorname{tr}(3+r)]}{[2 t(1-r)+r]\left[16 t(1-r)(1+\beta)-r \beta^{2}\right]} \\ \pi_{o}^{* N Y}=2 t\{r \beta-2 C(1-r)(1+\beta)[2 t(1-r)+r]+t(1-r)[(4-r)(1+\beta)-r]\}^{2} /[2 t(1-r)+r]\left[16 t(1-r)(1+\beta)-r \beta^{2}\right]^{2}\end{array}\right.$
Lemma 4. In the case of $N-Y$ model, with other parameters unchanged, when $1>t>r / 32(1-r)$, there is a unique equilibrium solution: $P_{h}^{* N Y}, P_{w}^{* N Y}, P_{o}^{* N Y}$, which make the profit function $\pi_{h}^{N Y}, \pi_{o}^{N Y}$ have the maximum value.

Corollary 4: With other parameters unchanged, when $1>t>r / 32(1-r)$, the optimal price of rooms in the hotel direct selling channel $P_{h}^{* N Y}$ is a decreasing function of $C$, The proof procedure is similar to the proof of Corollary 3.
Corollary 5: With other parameters unchanged, when $1>t>r / 32(1-r)$, the optimal wholesale price $P_{w}^{* N Y}$ is a decreasing function of $C$. The proof procedure is similar to the proof of Corollary 3. Corollary 6: With other parameters unchanged, when $1>t>r / 32(1-r)$ and $0<\beta<1 / 8$, the optimal price of OTA online sales platform rooms $P_{o}^{* N Y}$ is an increasing function of $C$, when $1>$ $t>r / 32(1-r)$ and $4 t(1-r) / r<\beta<1, P_{o}^{* N Y}$ is a decreasing function of $C$.

### 3.4 N-N Model

When both the hotel and the OTA cannot be canceled, customers have unperceived cost $\beta * P_{h}$ and $\beta * P_{o}$. At this point, the customer utility of direct channel is $U_{h}=V-(1+\beta) P_{h}-t(1-d)$, and the customer utility of the OTA channel is $U_{o}=V-(1+\beta) P_{o}-t d$. Therefore, the indifference point is $d^{*}=(1+\beta)\left(P_{h}-P_{o}+t / 2 t\right.$, according to the above assumptions, getting the profit functions of the hotel and the OTA:

$$
\left\{\begin{array}{c}
\left.\pi_{h}^{N N}=P_{h}^{N N} * r\left[(1+\beta)\left(P_{o}^{N N}-P_{h}^{N N}\right)+t\right] / 2 t+P_{w}^{N N} *\left\{r\left[(1+\beta) P_{h}^{N N}-P_{o}^{N N}\right)+t\right] / 2 t+(1-r)\left[1-(1+\beta) P_{o}^{N N}\right]\right\} \\
\left.\pi_{o}^{N N}=\left(P_{o}^{N N}-P_{w}^{N N}\right) *\left\{r\left[(1+\beta) P_{h}^{N N}-P_{o}^{N N}\right)+t\right] / 2 t+(1-r)\left[1-(1+\beta) P_{o}^{N N}\right]\right\}
\end{array}\right.
$$

Similar to Section 3.1, we can get the optimal price of the hotel and OTA as expressed:
the optimal price of the OTA channel $: P_{o}^{* N N}=[r(2+t r)+t(6-7 r)] / 4(1-r)(1+\beta)[2 t(1-r)+r]$;
the optimal price of the hotel direct channel: $P_{h}^{* N N}=[1+t(1-r)] / 2(1-r)(1+\beta)$;
the optimal price of the wholesale price: $P_{w}^{* N N}=1 / 2(1-r)(1+\beta)$;
In this case, the optimal $P_{h}^{* N N}, P_{w}^{* N N}$ and $P_{o}^{* N N}$ are inserted into the profit functions $\pi_{h}^{N N}$ and $\pi_{o}^{N N}$ respectively, and the profits of the hotel direct channel and the OTA are as follows:

$$
\left\{\begin{array}{c}
\pi_{h}^{* N N}=\left[4 t+4 r\left(1+t^{2}\right)-t^{2} r(r-4 t r+3+8 t)\right] / 16(1-r)(1+\beta)[2 t(1-r)+r] \\
\pi_{o}^{* N N}=t(2-r)^{2} / 32(1+\beta)[2 t(1-r)+r]
\end{array}\right.
$$

Lemma 5. In the case of $N-N$ model, with other parameters remain unchanged, the optimal price of the hotel direct room $P_{h}^{* N N}$ is the increasing function of $t$; the wholesale price $P_{w}^{* N N}$ and the hotel direct channel price $P_{h}^{* N N}$ are the decreasing function of $(1-r)$, the OTA channel price $P_{o}^{* N N}$ is the increasing function of $t$.

## 4. Hotel's Expected Revenue Ratio Analysis

This paper mainly studies the influence of the cancellation cost on the hotel cancellation strategy selection, to make the optimal pricing decision. We compare the expected profits of both parties under the different scenarios mentioned above, to determine the optimal cancellation and pricing strategies. Assuming that the probability of hotel direct channel allowing customers to cancel room booking and the probability of the OTA allowing customers to cancel room booking are both $1 / 2$, and the customer cancellation perceived cost coefficient $\beta=0.5$. Although such an assumption of probability parameter has limitations, it does not affect the discussion and analysis of the unit loss cost of unsubscription $C$ in this paper. $\pi_{h}^{Y}=\left(\pi_{h}^{* Y Y}+\pi_{h}^{* Y N}\right) / 2$ is the hotel direct channel allows customers to cancel room reservations. $\pi_{h}^{N}=\left(\pi_{h}^{* N Y}+\pi_{h}^{* N N}\right) / 2$ is that the hotel direct channel does not allow customers to cancel the room reservation expected profit revenue. In this paper, the physical hotel is the leader in the game decision, so this section only needs to discuss the optimal profit decision of the hotel according to different situations, and does not discuss the OTA's strategy.

As described in the model in Section 3, there are:

$$
\begin{gathered}
\pi_{h}^{Y}=\left\{4 r[1-C(1-r)]^{2}+4 r t^{2}(1-r)^{2}+t(1-r)[2-2 C(1-r)+r]^{2}\right\} / 32[2 t(1-r)+r]+ \\
\frac{3 C^{2} r(1-r)[4 t(1-r)+r]-C r\left[6 t r^{2}(4 t-1)+2 t(7+12 t)-r\left(48 t^{2}+8 t-1\right)\right]+t\left[8 t+3 t r^{3}(4 t-1)-r^{2}\left(24 t^{2}+11 t-1\right)+2 r\left(5+3 t+6 t^{2}\right)\right]}{[2 t(1-r)+r](96 t-96 t r-r)}
\end{gathered}
$$

$$
\begin{gathered}
\pi_{h}^{N}=\left[4 t+t r^{3}(4 t-1)-t r^{2}(3+8 t)+4 r\left(1+t^{2}\right)\right] / 48(1-r)[2 t(1-r)+r]+ \\
\frac{2\left\{\frac{3}{2} C^{2}(1-r)[2 t(1-r)+r]^{2}-c[2 t(1-r)+r]\left[\frac{1}{2} r+t(1-r)\left(6-\frac{5}{2} r\right)\right]+t\left[6 t+t r^{3}(4 t-1)+r\left(4 t^{2}-3 t+5\right)-\frac{1}{2} r^{2}\left(1+4 t+16 t^{2}\right)\right]\right\}}{(96 t-96 t r-r)[2 t(1-r)+r]}
\end{gathered}
$$

The size relationship between $\pi_{h}^{Y}$ and $\pi_{h}^{N}$ will now be analyzed by parameter values to determine the influence of $C, t$ and $r$ on the optimal cancellation and pricing strategies of the hotel.

### 4.1 Numerical simulation analysis

Because of the complexity of the model, this section uses the Matlab simulation system, set up different parameter values analyzes the influence of the hotel profits, numerical test mainly for the hotel allows the unit loss cost of unsubscription $C$, the cost of channel transfer for customers $t$ and the original market demand scale $r$ to the hotel in different cases of comparative analysis of expected profit benefit, to analyze the cancellation and pricing strategies of the hotel.

First, analyze the impact of the unit loss cost of unsubscription $C$ that the hotel allows to unsubscribe on the expected profit of the hotel in two cases: $t=0.5, r=0.5$, and $t=0.8, r=$ $0.5,(1>t>r / 32(1-r))$, then in Figure 1.


Figure 1. The impact of $\boldsymbol{C}$ on expected profits
Figure 1 shows that in the case of $t=0.5, r=0.5$ and $t=0.8, r=0.5$, the expected profit benefits of cancellation and non-cancellation booking are a decrement function of the unit loss cost of unsubscription $C(0<C<1)$. And there are thresholds $C^{\prime}=0.46$ and $C^{\prime \prime}=0.51$ make $\pi_{h}^{Y}=\pi_{h}^{N}$, it is say, when the unit loss cost of unsubscription $0<C<C^{\prime}$, the main decision maker of the hotel direct selling channel should choose the $Y$ strategy, when $C^{\prime}<C<1$, they should choose the $N$ strategy. Similarly, when the unit loss cost of unsubscription $0<C<C^{\prime \prime}$, the main decision maker of the hotel direct selling channel should choose the $Y$ strategy, when $C^{\prime \prime}<C<1$, they should choose the $N$ strategy.

Considering whether the hotel allows customers to cancel reservations, in order to analyze the impact of the unit cost of channel transfer for customers $t$ on the expected profit of the hotel in one groups of parameter values: $C=0.5, r=0.5$, see Figure 2 .


Figure 2. The effect of $\boldsymbol{t}$ on the expected profit

Figure 2 shows that the expected profit of hotel direct selling channels is not monotonous about the unit cost of channel transfer for customers $t$ under the parameter assignments: $C=0.5, r=0.5$. There is a threshold value $t^{\prime}=0.725$ making $\pi_{h}^{Y}=\pi_{h}^{N}$, then, when the unit cost of channel transfer for customers is $0<t<t^{\prime}$, the main decision maker of the hotel direct selling channel should choose the $N$ strategy, when the unit cost of channel transfer for customers $t^{\prime}<t<1$, they should choose the $Y$ strategy.

Considering whether the hotel direct channel allows customers to cancel reservations, in order to analyze the impact of the original market demand scale $r$ on the expected profit of the hotel direct channel, the parameter value is the unit loss cost of unsubscription and the unit cost of channel transfer for customers $C=0.5, t=0.5$, as shown in Figure 3.


Figure 3. The effect of $r$ on expected profit
Figure 3 shows that in the case of $C=0.5$, and $t=0.5$, the expected profit of the hotel direct selling channel is that the original market demand scale $r$ is an increase function. In addition, a threshold $r^{\prime}=0.56$ makes $\pi_{h}^{Y}=\pi_{h}^{N}$, that is, when the original market demand scale is $0<r<r^{\prime}$, the main decision maker of the hotel direct selling channel should choose the $N$ strategy, when $r^{\prime}<r<1$, they should choose the $Y$ strategy.

## 5. Summary

This paper constructs a game model of cooperation and competition between the hotel and the OTA, and respectively analyzes the impact of the unit loss cost of unsubscription, the unit cost of channel transfer for customers, and the OTA's monopoly market segment on the policy choice of whether the hotel channel allows consumers to cancel reservations. Research finding: (1) When the unit loss cost of unsubscription to the hotel is small, the hotel chooses to allow the customer to cancel the reservation strategy. At this time, the hotel room pricing increases with the increase of the unit loss cost of unsubscription and the unit cost of channel transfer for customers. (2) When the unit loss cost of unsubscription caused by the customer cancellation of the hotel is large, the hotel should choose not to allow the customer to cancel the reservation strategy. At this time, the hotel room pricing decreases with the increase of the unit loss cost of unsubscription. (3) When the unit cost of channel transfer by customers is relatively low, that is, when the customer has a strong ability to obtain the hotel information, and the cost of converting the hotel is relatively low, the hotel direct selling channel should choose the non-cancellable booking strategy, to reduce the change of the hotel room redundant amount. (4) When the unit cost of channel transfer for customers is high, that is, the ability of customers to obtain hotel information is weak, that is, when the cost of converting the hotel is high, the hotel direct selling channel should choose the cancellation booking strategy to meet the needs of consumers' travel changes, to attract customers. (5) When the original market demand scale is small, the hotel direct selling channel should choose the strategy that customers can not cancel the reservation. When the original market demand is large, the hotel direct selling channel should choose the strategy that customers can cancel the reservation.

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## Appendix

## Proof of Lemma 1

$P_{h}^{* Y Y}$ seek partial guidance for the unit loss cost of unsubscription $C$ and the unit cost of channel transfer for customers $t$ respectively, get $\partial P_{h}^{* Y Y} / \partial c=\partial P_{h}^{* Y Y} / \partial t=1 / 2>0$. Therefore, the hotel direct selling price $P_{h}^{* Y Y}$ is about the increasing function of $C$ and $t$. Similarly, $\partial P_{w}^{* Y Y} / \partial C=-1 / 2<0$, therefore, the wholesale price $P_{w}^{* Y Y}$ is a reduction function of the unit loss cost of unsubscription $C$. Similarly elicit $\partial P_{o}^{* Y Y} / \partial t=[2-2 C(1-r)-r] r / 4[2 t(1-r)+r]^{2}$, when $0<C<1-r / 2(1-r)$, get $\partial P_{o}^{* Y Y} / \partial t>0$, Therefore, $P_{o}^{* Y Y}$ is an increasing function of the unit cost of channel transfer for customers $t$. When $1-r / 2(1-r)<$ $C<1$, get $\partial P_{o}^{* Y Y} / \partial t<0$, At this point $P_{o}^{* Y Y}$ is a decrasing funtion of the unit cost of channel transfer for customers $t$.

## Proof of Lemma 2

$\pi_{h}^{* Y Y}$ seek partial guidance for the unit loss cost of unsubscription C , get $\partial \pi_{h}^{* Y Y} / \partial c=-$ $\left\{2 t(1-C)+r^{2}[2 C(1-t)-t]+r[2(1-C)+t(4 C-1)]\right\} /[8 t(1-r)+4 r]$, because $8 \mathrm{t}(1-\mathrm{r})+4 \mathrm{r}>0$, so the positive and negative nature of the partial derivative function: $\partial \pi_{h}^{* Y Y} / \partial c$ is determined by funtion $f=$ $2 t(1-C)+r^{2}[2 C(1-t)-t]+r[2(1-C)+t(4 C-1)] \quad$. Make $\quad f=0 \quad, \quad$ obtain $\quad C=$ $\left[t\left(2-r^{2}\right)+r(2-t)\right] /\{2 t(1-2 r)+2 r[1-r(1-t)]\}$, when $0<\mathrm{C}<\left[t\left(2-r^{2}\right)+r(2-t)\right] /\{2 t(1-2 r)+2 r[1-r(1-t)]\}$, get $\partial \pi_{h}^{* Y Y} / \partial c<0$, therefore, the profit of the hotel direct selling channel $\pi_{h}^{* Y Y}$ is a decreasing function of the unit loss cost of unsubscription $C$; when $\left[t\left(2-r^{2}\right)+r(2-t)\right] /\{2 t(1-2 r)+2 r[1-r(1-t)]\}<C<1$, get $\partial \pi_{h}^{* Y Y} / \partial c>$ 0 , at this time, the profit of the hotel direct selling channel $\pi_{h}^{* Y Y}$ is a increasing function of the unit loss cost of unsubscription C. Similarly, $\pi_{o}^{* Y Y}$ seek partial guidance for the unit loss cost of unsubscription $C$, get $\partial \pi_{o}^{* Y} / \partial c=-t(1-r)[2-2 C(1-r)-r] / 8[2 t(1-r)+r]$, because $8[2 t(1-r)+r]>0$, so the positive and negative nature of the partial derivative function: $\partial \pi_{o}^{* Y Y} / \partial c=-t(1-r)[2-2 C(1-r)-r] / 8[2 t(1-r)+r]$ is determined by funtion $f=[2-2 C(1-r)-r]$. Make $f=[2-2 C(1-r)-r]=0$ obtain $C=(2-r) / 2(1-r)$, when $0<C<$ $(2-r) / 2(1-r)$, get $\partial \pi_{o}^{* Y Y} / \partial c<0$, therefore the profit of the OTA $\pi_{o}^{* Y Y}$ is a decreasing function of the unit loss cost of unsubscription $C$; when $(2-r) / 2(1-r)<C<1$, get $\partial \pi_{0}^{* Y Y} / \partial \mathrm{c}>0$, at this time ,the profit of the OTA $\pi_{o}^{* Y Y}$ is a increasing function of the unit loss cost of unsubscription $C$.

## Proof of Lemma 3

The profit function of OTA sales platform $\pi_{o}^{Y N}$ seeks the partial derivative of $P_{o}^{Y N}$, get that $\partial \pi_{o}^{Y N} / \partial P_{o}^{Y N}=$ $\left\{P_{h}^{Y N} r-t r+P_{w}^{Y N}(1+\beta)[2 t(1-r)+r]-2 P_{o}^{Y N}(1+\beta)[2 t(1-r)+r]\right\} / 2 t ; \quad \partial^{2} \pi_{o}^{Y N} / \partial P_{o}^{Y N 2}=-(1+\beta)[2 t(1-r)+r] / t<0$. It is concluded that the profit function of OTA sales platform $\pi_{o}^{Y N}$ is a convex function of $P_{o}^{Y N}$ with maxima. Set $\partial \pi_{o}^{Y N} / \partial P_{o}^{Y N}=0$, then get $P_{o}^{* Y N}=P_{h} r-t r+P_{w}(1+\beta)[2 t(1-r)+r] / 2(1+\beta)[2 t(1-r)+r]$, replacing $P_{o}^{* Y N}$ into the profit function $\pi_{h}^{Y N}$ of the hotel direct selling channel seeks the second order partial derivative and the second order mixed partial derivative for $P_{h}^{Y N}, P_{w}^{Y N}$, and get the Hessian matrix:

$$
\left[\begin{array}{cc}
\frac{\partial^{2} \pi_{h}^{Y N}}{\partial P_{h}^{Y N}} & \frac{\partial^{2} \pi_{h}^{Y N}}{\partial P_{h}^{Y N} \partial P_{w}^{Y N}} \\
\frac{\partial^{2} \pi_{h}^{Y N}}{\partial P_{w}^{Y N} \partial P_{h}^{Y N}} & \frac{\partial^{2} \pi_{h}^{Y N}}{\partial P_{w}^{Y N}}
\end{array}\right]=\left[\begin{array}{cc}
-\frac{r[4 t(1-r)+r]}{2 t[2 t(1-r)+r]} & \frac{r(2+\beta)}{4 t} \\
\frac{r(2+\beta)}{4 t} & -\frac{(1+\beta)[2 t(1-r)+r]}{2 t}
\end{array}\right]=\frac{r\left[16 t(1-r)(1+\beta)-r \beta^{2}\right]}{16 t^{2}}
$$

Note $F(\beta)=16 t(1-r)(1+\beta)-r \beta^{2}, 0<\beta<1$, then $F(0)=16 t(1-r)>0, F(1)=32 t(1-r)-r$. Whether the matrix is positive definite is determined by the function $F(\beta)$. For $F(1)>0$, there is $t>r / 32(1-r)$, that is, $F(\beta)$ is more than 0 on $\beta[0,1]$, so the matrix is positive definite, so there is the only most balanced solution $P_{h}^{* Y N}, P_{w}^{* Y N}, P_{o}^{* Y N}$, so that the profit functions $\pi_{h}^{Y N}$ and $\pi_{o}^{Y N}$ have the maximum value.

## Proof of Corollary 1

According to the optimal direct channel price of $P_{h}^{* Y N}$, and then seek the partial guide of $C$, there are $\partial P_{h}^{* Y N} / \partial C=(1+\beta)[8 t(1-r)-r \beta] /\left[16 t(1-r)(1+\beta)-r \beta^{2}\right]$. According to Lemma 3, when $1>t>r / 32(1-r)$, the function $F(\beta)$ is greater than 0 on $\beta[0,1]$, at this time, the positive and negative nature of the partial derivative function: $\partial P_{h}^{* Y N} / \partial C=(1+\beta)[8 t(1-r)-r \beta] /\left[16 t(1-r)(1+\beta)-r \beta^{2}\right]$ is determined by $8 t(1-r)-r \beta$.

Because $8 t(1-r)-r \beta>r / 4-r \beta$, make $r / 4-r \beta=0$ then has $\beta=1 / 4$; so there is when $t>r / 32(1-r)$ and $0<\beta<1 / 4$, the partial derivative function $\partial P_{h}^{* Y N} / \partial C>0$, that is, $P_{h}^{* Y N}$ is an increasing function of the unit loss cost of unsubscription $C$. When $8 t(1-r)-r \beta<0$, there are $8 t(1-r) / r<\beta$, so when $1>t>r / 32(1-r)$ and $8 t(1-r) / r<\beta<1$, the partial derivative function $\partial P_{h}^{* Y N} / \partial C<0$, so $P_{h}^{* Y N}$ is a minus function of the unit loss cost of unsubscription $C$.

## Proof of Corollary 2

According to the selling price of the optimal online sales platform OTA: $P_{o}^{* Y N}$, then we find the partial derivative for $C$, with $\partial P_{h}^{* Y N} / \partial C=r[2 t(1-r)(2-\beta)-r \beta] /\left[16 t(1-r)(1+\beta)-r \beta^{2}\right][2 t(1-r)+r]$. According to Lemma 3: when $1>t>r / 32(1-r)$, The function $F(\beta)$ in $\beta[0,1]$ are all greater than 0 , at this time, the positive and negative properties of $\partial P_{h}^{* Y N} / \partial C=r[2 t(1-r)(2-\beta)-r \beta] /\left[16 t(1-r)(1+\beta)-r \beta^{2}\right][2 t(1-r)+r]$ is determined by function $2 t(1-r)(2-\beta)-r \beta$. And then $2 t(1-r)(2-\beta)-r \beta>r(2-\beta) / 16-r \beta$, make $r(2-\beta) / 16-r \beta=0$ has get $\beta=2 / 17$, therefore, when $1>t>r / 32(1-r)$ and $0<\beta<2 / 17$, then $\partial P_{o}^{* * N} / \partial C>0$, therefore, teh function $P_{o}^{* Y N}$ is an increasing function of the unit loss cost of unsubscription $C$. Order $2 t(1-r)(2-\beta)-$ $r \beta<0$, with $4 t(1-r) /[2 t(1-r)+r]<\beta$, therefore, when $1>t>r / 32(1-r)$ and $4 t(1-r) /[2 t(1-r)+r]<\beta<1$, then $\partial P_{o}^{* Y N} / \partial C<0$, therefore, the function $P_{o}^{* Y N}$ is a decreasing function of the unit loss cost of unsubscription $C$.

## Proof of Corollary 3

Since $\partial P_{w}^{* Y N} / \partial C=\left[4 \operatorname{tr} \beta(r-1)-r^{2}\right] /\left[16 t(1-r)(1+\beta)-r \beta^{2}\right][2 t(1-r)+r]$, when $1>t>r / 32(1-r)$, there is $4 \operatorname{tr} \beta(r-$ $1)-r^{2}<0$ and $\left[16 t(1-r)(1+\beta)-r \beta^{2}\right][2 t(1-r)+r]>0$, that is $\partial P_{w}^{* Y N} / \partial C<0$, then wholesale price $P_{w}^{* Y N}$ is a decreasing function of $C$.

## Proof of Lemma 4

The proof process is the same as the proof of Lemma 3.

## Proof of Corollary 6

According to Lemma 3, when $1>t>r / 32(1-r)$, the profit function has an equilibrium solution, and the $F(\beta)=16 t(1-r)(1+\beta)-r \beta^{2}$ is greater than 0 in the $\beta[0,1]$ interval. With the optimal selling price of OTA sales platform $P_{o}^{* N Y}$ for the unit loss cost of unsubscription $C$, there is $\partial P_{o}^{* N Y} / \partial C=$ $(1+\beta)[4 t(1-r)-r \beta] /\left[16 t(1-r)(1+\beta)-r \beta^{2}\right]$, of which $16 t(1-r)(1+\beta)-r \beta^{2}>0$, so the positive and negative nature of $\partial P_{o}^{* N Y} / \partial C$ is determined by $4 t(1-r)-r \beta$. And because $4 t(1-r)-r \beta>r / 8-r \beta$, set $r / 8-r \beta>0$, there is $1 / 8>\beta>0$. That is, when $1>t>r / 32(1-r)$ and $1 / 8>\beta>0$, then $\partial P_{o}^{* N Y} / \partial C=$ $(1+\beta)[4 t(1-r)-r \beta] /\left[16 t(1-r)(1+\beta)-r \beta^{2}\right]>0$, therefore, $P_{o}^{* N Y}$ is an increasing function of the unit loss cost of unsubscription $C$. Set $4 t(1-r)-r \beta<0$, there is $4 t(1-r) / r<\beta$, therefore, when $1>t>r / 32(1-r)$ and $4 t(1-r) / r<\beta<1$, function $P_{o}^{* N Y}$ is a decreasing function of the unit loss cost of unsubscription $C$.

## Proof of Lemma 5

According to the expression of the equilibrium solution $P_{h}^{* N N}, P_{w}^{* N N}$ and $P_{o}^{* N N}$, including: $\partial P_{h}^{* N N} / \partial t=$ $1 / 2(1+\beta) \quad ; \quad \partial P_{h}^{* N N} / \partial(1-r)=-1 / 2(1-r)^{2}(1+\beta) \quad ; \quad \partial P_{w}^{* N N} / \partial(1-r)=-1 / 2(1-r)^{2}(1+\beta) \quad ; \quad \partial P_{o}^{* N N} / \partial t=$ $(2-r) r / 4(1+\beta)[2 t(1-r)+r]^{2}$, then $\partial P_{h}^{* N N} / \partial t>0 ; \partial P_{h}^{* N N} / \partial(1-r)<0 ; \partial P_{w}^{* N N} / \partial(1-r)<0 ; \partial P_{o}^{* N N} / \partial t>0$, while the other parameters remain constant, Lemma 5 is true.

