The application of Kelly Criterion in lottery mathematical statistics

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Abstract. The paper will introduce the concept of Gambler's Ruin by analyzing the psychological analysis of gamblers, and then research the probability problem in lottery. Assuming that there are no other influencing factors, and the winning probability is known, we find the simple model and calculate and analyze the best bet amount, that is, the best strategy for buying lottery tickets by using Kelly Criterion. The Kelly Criterion is a valid formula for calculating the optimal bet amount. We need to use the Kelly Criterion to calculate the formula, use the drawing tool to graph the function and find the highest point, which is the maximum bet amount. Analyze and discuss the relationship between investment risk and return. Finally, the combination of the two explains the player's happiness. Use the Kelly Criterion to define gambler's happiness. To summarize the role of Kelly Criterion in the case of known lottery probabilities.

Keywords: Kelly Criterion; lottery; Invest; Optimal decision; Gambler's Ruin; Happiness.

1. Introduction

Since the appearance of gambling, it has become a popular entertainment. With the development of our lives, the forms of gambling have become more diversified and are no longer limited to offline casinos. There are also easy tickets to buy. Lottery machines have appeared in supermarkets large and small, attracting more people to participate [1, 2]. It attracts people not only the convenience of buying, but also the variety of gameplay and forms as well as the huge prize money. With the development of more and more Sports, Sports Betting has become a favorite sport. Playing big and small games is not only about tight scores but also about lottery tickets, which provides more fun for the spectators. Under such circumstances, there will inevitably be some loyal fans of the lottery.

According to Plöntzke et al. article "Forms of pathological gambling: empirical research on consumers behavior of sport betting and lottery participants" to the survey data "Of the 108 subjects, 33.3% of the sample met the diagnostic criteria for pathological sports betting. In addition, of the 22 sports betting subjects who played the lottery, 92% were diagnosed as pathological lottery gamblers" [3]. From such a high percentage, we can see the great attraction of lottery, which will lead to people's addiction. So, what makes people so addicted, and are there any techniques we can use to get the most out of it? Whether it's pure luck or some skill?

We'll first introduce the concept of Gambler's ruin to analyze it. The problem is discussed under the condition that there are no other influencing factors, and the fixed probability of winning is known. The so-called lottery it is not a hundred percent of the winning rate, and the mentality of gamblers is constantly with their own money to get a small chance of continuous winning rate, constantly on the bet [4, 5]. Let's look at the simple sports lottery example to illustrate better. For example, if two teams play each other, let's assume that, in the absence of external influence conditions, both teams are equally level and have a 50% chance of winning. Players bet on the winning team. Correctly guessing the winning team will earn three times the invest, and nothing for not guessing correctly. This is a simple "triple or nothing" model. For gamblers, triple is a great temptation, when they bet on the winning team, they can triple the reward. This gives them an incentive to put in more money and get a bigger amount. But if they didn't bet on the winning team, they would do the same again, repeating the bet until there was no money left. This is a cycle that never reaches the fulfillment of expectations. So, it makes them bet again and again. As Coolidge explained in his article "The Gambler's Ruin", "a gambler will raise his wagers when he wins and will not reduce his wagers when he loses, even if every wager has a positive expected value, he will eventually go bankrupt" [6]. So, to avoid this kind of problem, can we get a relatively reasonable betting amount to get a relatively large bonus?

As Lototsky, & Pollok, A., "Kelly Criterion provides A strategy that maximizes the long-term growth of winnings when the expected return of bets is positive" [7]. We will discuss how to maximize returns and find the best bet amount. Here we need to introduce the Kelly Criterion as a formula to mathematize the lottery problem. Then use the graphing tool to find the maximum value of the function, that is, the best bet amount. By analyzing and discussing Kelly Criterion's definition of happiness.

2. Method

The Kelly Formula is used to calculate the proportion of money an investor should put on each bet in order to get the maximum return under uncertain conditions. The purpose of the formula is to maximize long-term gains while guaranteeing not to go bankrupt [8, 9]. We used Graphing tool graphing function image, which is more intuitive to find the maximum function [10].

3. Results and discussion

3.1 solution for determining the amount of investment

Let's assume the player starts with M dollars. For this "triple or nothing" problem, assume that players invest "x" percent each time they buy a ticket. In an ideal situation, play two rounds, one with the winning team and one with the wrong team. So that after the first round, the player's amount is $(1-x)M+3xM=(1+2x)M$. After the second round, the player's amount is $(1+2\mathrm{x})\mathrm{M}(1-\mathrm{x})$. And that gives us the function $M'=(1+2x)(1-x)M=(-2x^2+x+1)M$.

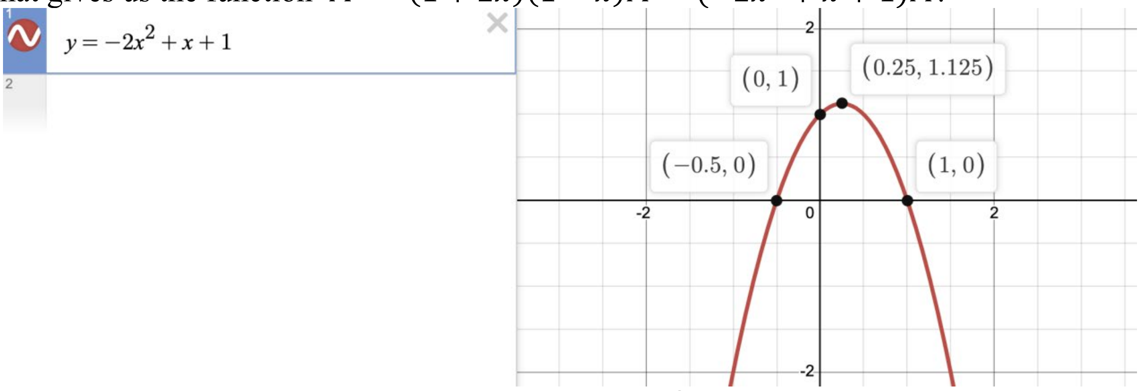

Fig 1. Function $\boldsymbol{y=-2x^2+x+1}$

We can see from the graph that we don't need to put in $x=0.25$ to maximize M'. So, we get $M'=(-2(0.25)^2+0.25+1)M=1.125M$. This is the maximum amount we can get. That's the Kelly Criterion. With Kelly Criterion we can find the optimal solution for determining the amount of investment. We also know from the graph that if$x\neq 0.25$, pick another point as invest will never be greater than $x=0.25$, there's a Gambler's ruin. So never do it.

3.2 maximize benefits using Kelly Criterion

But you know, the teams can't all be equal. To encourage people to buy lottery tickets. Lottery companies need to set different amounts to incentivize gamblers to buy. The following example illustrates this problem. Let's say we have two teams, A and B. Team A, who is worse than Team B, is only $\frac{1}{4}$ more likely to win than Team B, Team B is the better team and has a $\frac{3}{4}$ chance of winning. The rules of the lottery are that if you choose team A to win, and team A wins the game, you get three times the invest. If you choose team B to win, and team B wins the game, then you will get double invest. If the chosen team doesn't win, it gets nothing. It's not a "Triple or Nothing" or "Double or Nothing" question. Because the probability of winning is different between the two teams, so the probability of "Triple, Double or Nothing" are different. We first need to know the probability between the three of them. We know from the Winning probability:

$$Triple: Double: Nothing = \left(\frac{1}{4}\right) : \left(\frac{3}{4}\right) : \left(\frac{1}{4} + \frac{3}{4}\right) = 1:3:4$$

So that's what we get:

$$M' = (1 - x + 3x)(1 - x + 2x)^3(1 - x)^4 M = (1 + 2x)(1 + x)^3(1 - x)^4 M$$

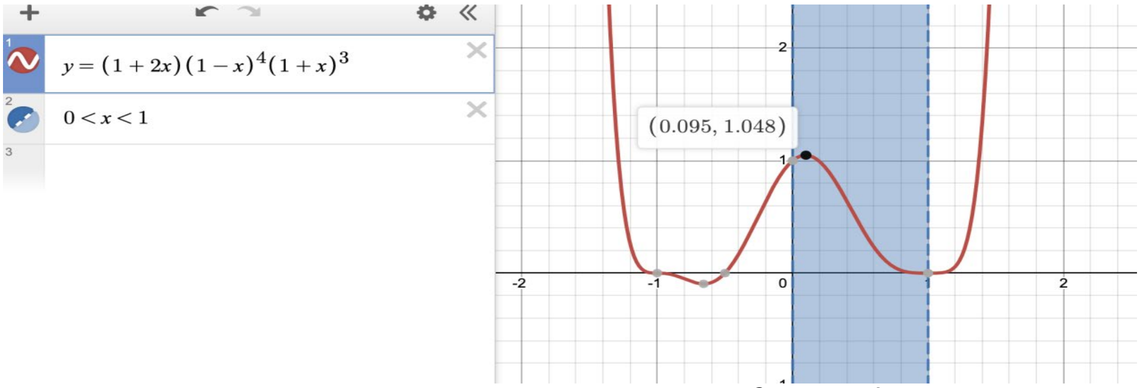

Fig 2. Function $\boldsymbol{y = (1 + 2x)(1 + x)^3(1 - x)^4}$

And here we must pay attention to the range of x. It is the fraction invested each round. So that $0 < x < 1$. So, we can find the maximum amount when $x = 0.095$. And $M' = (1 + 2 * 0.095)(1 + 0.095)^3(1 - 0.095)^4 M \approx 1.048M$.

By calculation we found that the amount we might get was very small. Real-life sports centers may offer larger sums, but they also involve greater investment risk. We've shown you how much money you can invest to maximize benefits using Kelly Criterion. What other applications can we have.

3.3 Kelly Criterion defines happiness

Kelly Criterion also defines happiness [11, 12]. The expression for this is $H = \log(M)$. That is $Happiness = \log(Money)$. The gambler gets a different amount of happiness each time the result is different. Take our first example above, "Triple or Nothing." For the first round $M_1 = (1 + 2x)M$. So that $H_1 = \log(1 + 2x)$. For the second round $M_2 = (1 - x)M$. So that $H_2 = \log(1 - x)$.

And by M_1 and M_2 we can find the average of H. $\bar{H} = \frac{1}{2}(\log(1 + 2x) + \log(1 - x)) = \frac{1}{2}\log((1 + 2x)(1 - x))$.

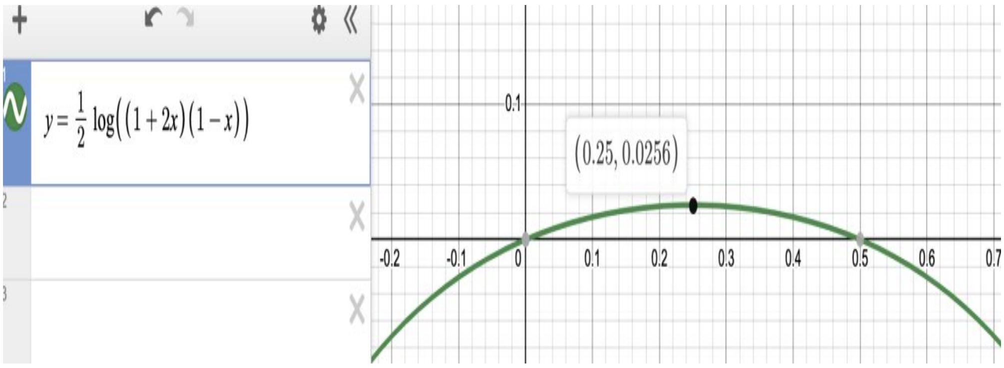

Fig 3. Function $\boldsymbol{y = \frac{1}{2}\log((1 + 2x)(1 - x))}$

Happiness is highest when $x = 0.25$. That's the same thing as what we just calculated. This corresponds to the maximum amount of money we're talking about.

4. Conclusion

Therefore, Kelly Criterion can explain and predict investment problems well. But there is a certain premise to the question. The first is a relatively fair game. There are all sorts of things that can happen in a game match, but we're using Kelly Criterion to assume that there's no external interference. Second, we need to know the specific probability of winning to predict. For example, the team's winning probability, but in the actual situation, we may not be able to perfectly predict the team's winning probability, and the team's winning probability is not certain. Our example is only carried out under ideal conditions. This makes it possible that Kelly Criterion's prediction of lottery problems in everyday life may not be that accurate. Besides, for example, MacLean et al., pointed out in the article " Good and bad properties of the Kelly criterion", "The main drawback of Kelly Criterion is that the suggested stakes can be high" [13].

So, in the real world, Kelly Criterion can be a good reference tool when it comes to the many factors involved in the sport lottery, and it can't be a one hundred percent predictor. Gambling is a project where it is possible to get many times the amount for a small amount of money, but it is also a multi-factor and risky project. Otherwise, there wouldn't be Gambler's Ruin.

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