ISSN:2790-167X DOI: 10.56028/aehssr.1.1.154

A Study on Systematic Risks of U.S. and China Stock Markets Based on Markov Copula

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Abstract. In this paper, Markov SJC copula model is constructed based on the daily data of standard & Poor's index and Shanghai Shenzhen 300 index, and the systematic risk of American and Chinese stock market is empirically analyzed. The results show that SJC copula can well depict the systematic risk of American and Chinese stock market, the risk dependence has obvious tail asymmetry characteristics, and the probability of low risk dependence is higher than that of high risk dependence.

Keywords: Systematic risk; Markov mechanism transformation; SJC Copula model.

1. Introduction

With the rapid development of economic globalization, the continuous changes of financial system, and the informatization and networking of financial activities, the correlation within the financial system and between the financial system and the real economy is closer. This change not only improves the efficiency of resource allocation, but also potentially changes the nature of financial risk. The American subprime crisis in 2008 is a typical fact of the modern financial crisis, which has a wider scope and a greater impact on economic and social welfare. Systemic financial risk is an important source of risk that triggered the crisis.

Traditional static Copula models assume that the correlation parameters are constants and cannot capture the dynamic correlation structure between variables. In order to overcome this problem, scholars have proposed many dynamic Copula models in recent years, such as time-varying Copula model, semi-parametric dynamic Copula model, random Copula model and Markov mechanism transformation Copula model. Manner et al. reviewed these dynamic time-varying Copula models. The research shows that the Markov mechanism transforms the Copula model has a better data fitting effect than other dynamic Copula models[1].

In view of the superiority of Markov mechanism conversion Copula model in theory and practice, and in order to make up for the lack of scholars' research on the systematic risk of stock market. This paper examines the tail correlation structure of the volatility of the US and Chinese stock markets under extreme market conditions, select the appropriate Copula function, and constructs the corresponding Markov mechanism to transform the Copula model. At the same time, the systemic risk of the US and Chinese stock markets are studied, and the asymmetric and tail dynamic characteristics of risk dependence may be examined[2].

2. Markov Mechanism Transformation Copula Model

The Markov mechanism transformation Copula model is constructed to describe the systematic volatility of stocks, making x_t the volatility of stock returns. The systematic volatility is characterized by the joint distribution function $H(x_t, x_{t-1})$ of continuous volatility variables x_t and x_{t-1} at times t and $t-1$ [3].

First, construct a Markov mechanism to transform the Copula model to describe the systematic fluctuation of stocks. Let x_t be the volatility of stock returns. The systematic fluctuation is the joint

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distribution function of the continuous volatility variables x_t and $x_{t\text{-}1}$ at time t and $t-1$ $H(x_t, x_{t-1})$ characterizes. According to Sklar's theorem, there exists a Copula $C(\cdot,\cdot):[0,1]^2 \to [0,1]$, which makes:

$$H(x_t, x_{t-1}) = C(u_1, u_2 \mid \theta) \tag{1}$$

Where $u_1 = F(x_t)$ and $u_2 = F(x_{t\text{-}1})$ are marginal distribution functions of x_t and $x_{t\text{-}1}$ respectively; θ is the parameter vector of Copula. It can be seen that Copula is a joint distribution function constructed by the marginal distributions $u_1 = F(x_t)$ and $u_2 = F(x_{t\text{-}1})$ uniformly distributed in the interval [0,1], which fully captures the correlation between continuous volatility variables x_t and $x_{t\text{-}1}$. Copula can be used to conveniently measure the tail correlation of two variables in extreme market situations, that is, the probability that the two variables are at the lower (left) tail or the upper (right) tail simultaneously. The lower and upper tail correlation coefficients of continuous volatility variables x_t and $x_{t\text{-}1}$ are:

$$\lambda^L = \lim_{x\to 0^+} \Pr[F(x_t) \le u \mid F(x_{t-1}) \le u] = \lim_{x\to 0^+} \frac{C(u,u \mid \theta)}{u} \tag{2}$$

$$\lambda^U = \lim_{x\to 1^-} \Pr[F(x_t) > u \mid F(x_{t-1}) > u] = \lim_{x\to 1^-} \frac{1-2u+C(u,u \mid \theta)}{1-u} \tag{3}$$

In the formula λ^L, $\lambda^U \in [0,1]$. If $\lambda^L(\lambda^U) \in (0,1]$, x_t and $x_{t\text{-}1}$ do not have lower (upper) tail correlation. Different tail correlation structures can be characterized by selecting different Copula functions.

Considering that continuous volatility variables may have both lower and upper tail correlations and exhibit asymmetric characteristics. Therefore, SJC Copula is introduced, which can capture the correlation of lower tail and upper tail simultaneously. SJC Copula is obtained by modifying "BB7"Copula (also known as Joe-Clayton Copula) [4]. SJC Copula is a very flexible Copula because it allows asymmetric lower tail and upper tail correlation and includes symmetric tail correlation as a special case. SJC Copula is expressed as:

$$C_{SJC}(u_1,u_2 \mid \lambda^U,\lambda^L) = 0.5(C_{JC}(u_1,u_2 \mid \lambda^U,\lambda^L) + C_{JC}(1-u_1,1-u_2 \mid \lambda^L,\lambda^U) + u_1 + u_2 - 1 \tag{4}$$

Where $C_{JC}(u_1,u_2 \mid \lambda^U,\lambda^L)$ is BB7 Copula, defined as:

$$C_{JC}(u_1,u_2 \mid \lambda^U,\lambda^L)=1-(1-\{[1-(1-u_1)^k]^{-r}+[1-(1-u_2)^k]^{-r}-1\}^{-1/r})^{1/k} \tag{5}$$

Where, $k = 1/\mathrm{lb}(2-\lambda^U)$, $r = -1/\mathrm{lb}(\lambda^L)$, $\lambda^L \in (0,1)$, $\lambda^U \in (0,1)$. When $\lambda^L{=}\lambda^U$, SJC Copula is symmetric. Another Copula that can capture the correlation between lower tail and upper tail at the same time is hybrid Copula. The following two mixed Copula are constructed: Gumbel mixed Copula and Clayton mixed copula:

$$C_{GM}(u_1,u_2 \mid \theta) = \omega C_{SG}(u_1,u_2 \mid \alpha_1) + (1\text{-}\omega)\; C_{Gum}(u_1,u_2 \mid \alpha_2) \tag{6}$$

$$C_{CM}(u_1,u_2 \mid \theta) = \omega C_{Clay}(u_1,u_2 \mid \alpha_1) + (1\text{-}\omega)\; C_{SG}(u_1,u_2 \mid \alpha_2) \tag{7}$$

Assuming that the above-mentioned traditional static Copula correlation parameters do not change with time, the tail dynamic correlation that may exist between continuous volatility variables cannot be captured. Therefore, it is necessary to extend the static Copula model to the dynamic Copula model. This paper uses Markov mechanism in dynamic Copula model to transform Copula model[5]. The state variable s_t is introduced into Copula function, assuming $s_t = \{0,1\}$ follows a first-order two-state Markov process, and the state transition probability is set $(u_{1t},u_{2t} \mid s_t = i) \sim C(u_{1t},u_{2t} \mid \theta_i)$, $i = 0,1$. Therefore, Copula parameters change with the change of state variable s_t , which can capture risk-dependent tail dynamics.

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3. Empirical research

3.1 Data and descriptive statistics

In this paper, the volatility data of S & P index (SP500), Shanghai and Shenzhen 300 index (HS300) and index daily return are used as the research samples. The time span of the data is from January 4, 2002 to November 29, 2019. By eliminating the inconsistent data on the trading day and making the trading day match, the final sample data is 4206 groups. All data are from Wind database. First, take logarithmic difference processing for data, that is, $r_t = \ln P_t - \ln P_{t-1}$, P is closing price. The volatility of return rate adopts the method of return sequence variance. The formula of variance method of income series is as follows:

$$\hat{\sigma}^2 \triangleq \frac{1}{N-1}\sum_{n=1}^{N}(r_{t-n}-\bar{r})^2 \qquad \bar{r} = \frac{1}{N}\sum_{n=1}^{N} r_{t-n} \tag{7}$$

In the formula, r_{t-n} is the income sequence, $\hat{\sigma}^2$ is the unbiased estimation of σ^2 , $\bar{r}$ is the mean value of the income sequence of N samples, and r_{t-n} is the daily return rate of samples. In the above formula, volatility can be measured according to standard deviation. At the same time, the sample variance method of return rate is also a generally accepted method to measure risk.

Table 1 Descriptive statistical results of yield volatility series

	Average value	Standard deviation	Skewness	Kurtosis	J-B	LM	ADF	BDS
SP500	0.0021	0.0114	0.2400	13.5134	25677.2*	80.71*	-67.14*	0.2045*
HS300	0.0033	0.0259	0.5576	11.3665	17796.8*	77.11*	-64.18*	0.2011*

*Represents rejection of J-B(Jarque-Bera) test, ARCH LM test, ADF unit root test or nonlinear BDS test at 5% significance level.

Table 1 gives descriptive statistics of the volatility of the two indices. As can be seen from table 1, the volatility of both indices is positively biased (skewness > 0), and there are leptokurtosis and fat-tail characteristics (kurtosis > 11), and both reject the assumption of normal distribution (Jarque-Bera statistics are significant).

3.2 Markov SJC Copula parameter estimation results

In order to obtain the possible tail dynamic characteristics of risk dependence in the U.S. and China's stock markets, SJC copula with the best fitting effect is selected, and the corresponding Markov mechanism transformation copula model is constructed for analysis.

As can be seen from figs.1 the risk dependence of the U.S. and Chinese stock markets shows obvious tail dynamic characteristics. At the same time, the persistence probability of regime0 is much smaller than that of regime1, which indicates that the US and Chinese stock markets are in a low-risk dependency regime in most periods.

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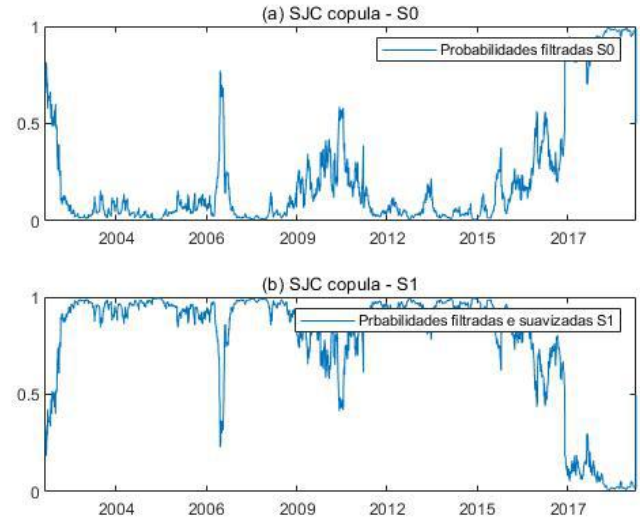

Fig. 1 Smoothing Probability of High Risk Dependent regime(S0) and Low Risk Dependent regime(S1)

Figure 1 is a continuous conditional probability diagram of two stock markets in a state of high and low risks dependence. The upper figure is a high-risk dependent regime probability map, and the lower figure is a low-risk dependent regime probability map. It can be seen from the figure that the duration of the low-risk dependency state is more lasting. When the stock market risk dependency between the United States and China is at regime0, the two have higher risk dependency, so the probability of financial systemic risk is higher. As can be seen from fig. 2(a), the probability of the initial state being regime0 is close to 1, which indicates that the sino-us stock market is in a state of high risk dependence at the initial time. Since April 2006, the high-risk dependence probability of U.S. and Chinese stock markets began to increase, then gradually changed from high-dependence regime to low-dependence regime, and again changed to high-dependence regime in January 2009, and remained in a high-dependence state for a long time. This is because the financial crisis that broke out in 2008 intensified the linkage between global stock markets. After the outbreak of the financial crisis, the systemic risks between the U.S. and Chinese stock markets have increased significantly, which also confirms that the global financial turmoil triggered by the U.S. subprime mortgage crisis has caused structural changes in the correlation of stock markets. Since 2017, the trade friction between the United States and China has increased their risk dependency. Since 2017, the stock markets of the United States and China have shown extremely high risk dependency.

4. Conclusions

This paper constructs the copula model of Markov mechanism transformation to study the systematic risk of American and Chinese stock markets. This paper selects the daily data of S & P and CSI 300 index, calculates the volatility of stock return as the proxy variable of stock risk, constructs Markov SJC copula model, and makes an empirical analysis on the systematic risk of stock market in the United States and China. The results show that SJC copula function can well describe the systematic risk of American and Chinese stock market, the risk dependence has obvious tail asymmetry, and the probability of low risk dependence is higher than high risk dependence. In addition, the research based on Markov mechanism transformation SJC Copula model shows that the risk dependence of the U.S. and China stock markets also shows significant tail dynamic characteristics.

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5. Acknowledgements

Thanks to the research project of General project of University-level scientific Research Fund of Xi 'an Aeronautical University (2020KY2233); Higher Education Research Project of Xi 'an Aeronautical University (2021GJ1009); Higher Education Scientific research project of Shaanxi Province(XGH21232).

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