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Preliminary Attempt to Implement Quality Education in Mathematics Teaching

Youlin Liu*, Fandong Xiang

Hunan Sany Polytechnic College Changsha, Hunan, China

e-mail: xnxyliuyl@126.com

Abstract. This paper discusses several ways to implement quality education in mathematics teaching from the aspects of paying attention to the teaching process of knowledge occurrence, strengthening the guidance of learning method, cultivating students' spirit of conscious exploration, and improving students' mathematical quality with transforming ideas.

Keywords: mathematics; knowledge; quality education; method

The core of quality education is to improve the quality of students. The mathematics quality of students mainly includes four aspects: exponential knowledge, mathematical skills, mathematical ability and mathematical thinking method. Mathematical knowledge is the foundation.Skill refers to the correct use of knowledge, through practice and acquired after the operation of solving problems.Ability is the activation of knowledge and precise and skilled skill.Mathematical thinking method is an important means to acquire these knowledge and skills. It is an important task for mathematics educators to improve students' mathematical quality by transforming their thoughts, attaching equal importance to knowledge and ability. Therefore, I have made some preliminary explorations in teaching practice.

1. Pay attention to the transformation of ideas and deepen the through understanding of knowledge.

Mathematics is a science with rigorous logic. The description of definitions, equations, theorems and laws in textbooks is very precise and rigorous. The students do find it difficult to understand. At this time, if the teacher can use the transformation of thought, timely guide students to try to find out the equivalent proposition after the change, after the transformation of the formula, so as to deepen the thorough understanding of knowledge, and can cultivate students' consciousness of transformation, the spirit of thinking.

The definition of odd and even function is a new and tricky concept for students, we use its equivalent definition to explain, so that students can understand more deeply and more thoroughly.That is, if f (x) - f (-x) = 0 for any X in the symmetric interval (-a, a), then f (x) is an even function; If f (x) + f (-x) = 0, then f (x) is an odd function.

Example 1: Determine the parity of function of $f(x) = \lg(\sqrt{x^2+1}+x)$

Solution: Obviously, the domain of f(x) is the fixed number set R, and

$$
\begin{aligned}
f(x)+f(-x) &= \lg(\sqrt{x^2+1}+x)+\lg(\sqrt{(-x)^2+1}-x) \\
&= \lg[\sqrt{(x^2+1)^2}-x^2] \\
&= \lg 1 = 0
\end{aligned}
$$

So $f(x) = \log(\sqrt{x^2+1}+x)$ is an odd function

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2. Use transformation thought to reveal the inner connection between knowledge.

Mathematics, as a science studying the relationship between spatial form and quantity in the real world, contains abundant dialectical thinking. When it is difficult to study A, we should adjust our thinking to study B, which is closely related to A. Through the discussion of B and the organic combination of A and B, we can achieve the purpose of deepening theme A.

Example 2: Assuming a, b and c are real numbers, and $\cos 2x = a\sin^2 x + b\sin x + c$ is identities, calculate the value of $a^2 + b^2 + c^2$.

Analysis: If we change the double angle into a single angle, and then discuss the value of each coefficient according to the meaning of identity, the problem can be solved, but it is very laborious and not concise. with the known condition $\cos 2x = a\sin^2 x + b\sin x + c$ is an identity, considering the relationship between special and general, we can make $x = 0$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{2}$, and turn the problem into finding the result of this ternary system of first-order equations:

$$\begin{cases} c = 1 \\ \dfrac{a}{4} + \dfrac{b}{2} = \dfrac{1}{2} \\ a + b + c = -1 \end{cases}$$

(Even students in junior high school can solve this question easily)，and we get the answer that $a = -2, b = 0, c = 1$, so $a^2 + b^2 + c^2 = (-2)^2 + 0 + 1^2 = 5$.

C. A. Yanofskaya, a professor at Moscow University, made a famous saying on "problem solving" that "problem solving means transforming the problem to be solved into a problem that has been solved." It is emphasized that transformation plays a leverage role in solving mathematical problems.

Example 3: Assume $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, m \in R^+$ and meet: $a_1 + b_2 = a_2 + b_3 = a_3 + b_4 = a_4 + b_1 = m$

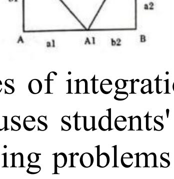
To prove：$a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4 < 2m^2$

Many students find it very difficult to solve this problem. After investigating the reason, they find that their thinking stays in the algebraic circle, so I suggested in my teaching: although it is an algebraic problem, can it be solved by geometric methods? Soon some students thought of the area formula and gave the following answer.

Proof: Because $a_1 + b_2 = a_2 + b_3 = a_3 + b_4 = a_4 + b_1 = m$, so we can draw a square ABCD with m as the side length and a quadrilateral $A_1B_1C_1D_1$, as shown in illustration, obviously：

$$S_{\Delta AA_1D_1} + S_{\Delta B_1BA_1} + S_{\Delta CC_1B_1} + S_{\Delta DD_1C_1} < S_{ABCD}$$

$$\text{e.:}\ \frac{1}{2}a_1b_1 + \frac{1}{2}a_2b_2 + \frac{1}{2}a_3b_3 + \frac{1}{2}a_4b_4 < m^2$$



The thought method of the above two cases of integration and transformation not only reveals the inner connection between knowledge, arouses students' interest in learning, but also cultivates students' good psychological quality of analyzing problems with a scientific attitude and improves students' ability to solve problems. At the same time, through the combination and transformation

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of number and form, the students naturally receive the education of transformation thought and improve their own mathematical quality.

3. Grasp the transformation of formula structure and train students' problem-solving skills.

Dialectical materialism holds that everything is always in constant change and movement. The methods and means used in mathematics teaching should "love the new and loathe the old" and promote innovation. We should often train students to solve practical problems with the concept of sports, transformed ideas and methods in classroom teaching. For example, in the chapter of "Trigonometric Function", in view of its characteristics of "Many formulas and complex subject structure, so it has strong flexibility in solving problems", in addition to teaching the "positive use" (i.e. from the left to the right of the formula) and "reverse use" (i.e. from the right to the left of the formula), it is more important to be good at grasping the formula structure according to the known conditions and the characteristics of the problem to be solved, Form transformation to cultivate students' skills in simplifying, evaluating and proving trigonometric equations. This is what we usually call the variation of the formula.

In the teaching of tangent formula of two corner sum

$$(\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha+\tan\beta})$$

we break the inherent form of formula and convert the formula into:

$$\tan\alpha+\tan\beta+\tan(\alpha+\beta)\tan\alpha\tan\beta=\tan(\alpha+\beta)$$

Related exercises:

To proof: $\tan 20°+\tan 40°+\sqrt{3}\tan 20°\tan 40°=\sqrt{3}$

Known $\alpha+\beta=\frac{\pi}{4}$, to proof: $(1+\tan\alpha)(1+\tan\beta)=2$

Make corresponding skill training. In this way, we can really get twice the result with half the effort in cultivating students' problem-solving skills.

Example 4: In ΔABC, to proof:

$$\sin^2A+\sin^2B-\sin^2C=2\sin A\sin B\cos C$$

Analysis: Pay attention to the sine theorem

Proof:

$$\because \sin^2A+\sin^2B-\sin^2C=\left(\frac{a}{2R}\right)^2+\left(\frac{b}{2R}\right)^2-\left(\frac{c}{2R}\right)^2$$
$$=\frac{1}{4R^2}(a^2+b^2-c^2)$$

$$2\sin A\sin B\cos C=2\times\frac{a}{2R}\times\frac{b}{2R}\times\frac{a^2+b^2-c^2}{2ab}$$
$$=\frac{1}{4R^2}(a^2+b^2-c^2)$$

$$\therefore \sin^2A+\sin^2B-\sin^2C=2\sin A\sin B\cos C$$

If we use pure triangular knowledge to prove it, it will be very complicated and easy to make mistakes. This proof pays attention to the idea of transformation, and is easily proved by sine quantification and cosine theorem. In my opinion, if the formula is always rigid in teaching, without deformation and transformation, it will undoubtedly inhibit students' creative thinking. Over time, it

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will become a huge obstacle to improving students' mathematical quality. These are worthy of sincere thoughts by educators.

4. Pay attention to the teaching of knowledge occurrence process and improve students' understanding of knowledge.

The author believes that in order to implement quality education in mathematics classroom teaching, we must establish a modern concept of mathematics teaching, adhere to the teacher as the leading role, give full play to the active role of students, thoroughly understand the heuristic principle from beginning to end, and pay full attention to the teaching of Knowledge Occurrence process, so as to comprehensively improve students' ability to understand knowledge.

The process of knowledge generation refers to the process of revealing and establishing the connection between old and new knowledge. In terms of specific mathematics classroom teaching, it is necessary to pay attention to mathematical concepts, rules, theorems, formulate and formation process, the exploration process of problem-solving ideas, problem-solving methods and rules of thinking, induction and generalization process.If classroom teaching can enable students to spread their thinking wings in these processes, they will fly freely in the ocean of mathematical knowledge, so as to acquire knowledge and improve their ability.

Lesson 1: Teach the concept of Geometric Progression.

The whole process of class teaching can be carried out in the form of teacher-student dialogue as follows:

Teacher: Please review how Arithmetic Progression is defined.

Student 1: If a sequence starts from item 2 and the difference between each item and its previous item is equal to the same constant, the sequence is called Arithmetic Progression.

$$a_n - a_{n-1} = d \quad (n = 2、3、4……)$$

Teacher: Great! Then please look at the following sequence:

$$3、9、27、81……$$

$$1、-\frac{1}{2}、\frac{1}{4}、-\frac{1}{8}……$$

What are the characteristics? Are they still arithmetic sequences?

Student 2: Obviously, these two sequences are not arithmetic sequences, their characteristics seem to be……

Student 3: I found the following mathematical laws:

$$\frac{9}{3} = \frac{27}{9} = \frac{81}{27} = \cdots = 3$$

$$\frac{\frac{-1}{2}}{1} = \frac{\frac{1}{4}}{\frac{-1}{2}} = \frac{\frac{-1}{8}}{\frac{1}{4}} = \cdots = -\frac{1}{2}$$

Teacher: Wonderful! Now can we imitate the arithmetic sequence to describe the common characteristics of the two sequences?

Student 4: Starting from term 2, the quotient of each term divided by its previous term equals the same constant.

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Teacher: Well, what should this sequence be called? Can you express it completely?

Student 5: If a sequence starts from item 2 and the ratio between each item and its previous item is equal to the same constant, the sequence is called Geometric Progression, and the constant is called common ratio.

Teacher: Can you write it like Arithmetic Progression?

Student 6: Suppose there has a sequence $\{a_n\}$, $\frac{a_n}{a_{n-1}}$ =q（常数）, (n=2、3、4……), then we called $\{a_n\}$ as Geometric Progression, "q" is the common ratio.

Teacher: Great job! Please judge whether the following sequences are Geometric Progression.

3、3、3、3……

x、x、x、x……

Student 7: They are all Geometric Progression.

Teacher: Was he right?

Student 8: His judgment on the second sequence is incomplete. When x=0， it is not a geometric progression; while x≠0, it is the geometric progression.

Teacher：So what are the restrictions on the items and common ratio of Geometric Progression?

Student 9：$n \in N$ $a_n \neq 0$，"q" belongs to nonzero constant.

In the whole process, students are actively participating in, observing and thinking. It can be said that the concept of "Geometric Sequence" should be unforgettable to them.

Holland mathematics educator H·Freudenthal thinks that in mathematics science, teachers must deal with the materials reflected in the teaching books actively and according to the so-called Socratic re-creation method; at the same time, he emphasizes the value of teaching is not limited to remember the past knowledge, but also develop thinking training and improve students' understanding level and ability of knowledge. Therefore, in teaching practice, I make full use of the content of teaching materials, design the teaching process in line with students' cognitive structure and show my psychological activity process of solving problems, such as how to analyze, how to infer, how to choose methods and so on, helping students understand the materials and concepts so that they can truly reveal the essential attributes of mathematical concepts, formulas and theorems through their understanding of the process, so as to achieve the purpose of improving mathematical quality.

5. Trengthens the guidance of learning methods and teach students in accordance with their aptitude.

Quality education advocates that teaching should face all students, which is undoubtedly correct. However, in practical teaching, it is difficult to cover every student. Therefore, I try to divide students into good and poor categories or good, medium and poor categories according to their grades. In classroom teaching, firstly I give them collective lectures, and then put forward requirements for them according to categories, strengthening the guidance of learning methods, implementing teaching students according to their aptitude, and fully mobilizing students' learning enthusiasm.

A. Less but great classroom group teaching.

Teachers should carefully read the teaching syllabus for each lesson, study the teaching materials, deeply understand the students and design teaching plans suitable for the actual situation of the students, so as to achieve less and better (only a small part of time) based on the level of most students in class collective teaching, and leave most of the time for students to self-study, thinking and practice.

Lesson 2: Teach the concept of Magnitude.

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Teachers can analyze and explain it according to the following four levels.

① Geometric interpretation:

Complex number z=a+bi(a,b∈R) can be represented vector $\overrightarrow{OZ}$, the magnitude |z| is the length of $\overrightarrow{OZ}$, so |z| ≥0.

|z| is the distance from the complex number z corresponding to the point Z on the complex plane to the origin. As shown in the figure.

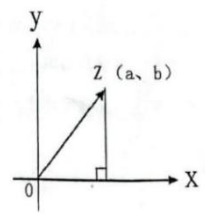

Figure2.

② Direct explanation:$|z| = \sqrt{a^2 + b^2}$, obviously|z|is a non-negative real number.

③ Analogy analysis: The concept of magnitude is a generalization of the concept of absolute value of real numbers. We can make the following analysis.

Table i.

The set of real number R	The set of complex number C
$\lvert x\rvert^2 = x^2$ $\lvert x\rvert = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$	$\lvert z\rvert^2 = z \cdot \bar{z}$ $\lvert z\rvert^2 \neq z^2$ z=a+bi (a,b∈R) $\lvert z\rvert = \sqrt{a^2 + b^2}$

④ Example explanation:

Find the magnitude of z=3+4i

z∈C, what is the set of points Z satisfying $2 < |z| \leq 3$

(The solution is omitted.)

Practice has proved that according to this model, only 4% of students require additional examples, while about 96% of students are very satisfied with the teaching effect. This shows that less but great classroom group teaching is feasible.

B. Teach students according to their aptitude and give them classified guidance.

To teach students in accordance with their aptitude, we must implement a principle. That is, if we want students to do something, we should guide them to do it by themselves. We should fully estimate their ability and guide them to overcome the difficulties in the process of knowledge generation tenaciously and persistently. And when students can not do things, they can not be asked to do. As long as teachers are serious and responsible for each student, and each student can fully develop their potential, improve ability and make progress, we teachers are successful!

Lesson 3: Learn the guidance for middle and upper students through learning Hyperbola.

The steps are as follows:

Students learn the hyperbola section in the textbook by themselves.

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Teachers give students guidance: In terms of knowledge structure, hyperbola and ellipse have internal connection and comparability. (From their definition, standard equations to their geometric properties.)

Re-understand ellipse and hyperbola with the help of eccentricity.

Do exercises. (Omit specific exercises.)

Lesson 4: Learn the guidance for poor students through learning Inequality of arithmetic and geometric means $\frac{a+b}{2} \geq \sqrt{ab}$

Guidance requirements and steps:

On the basis of listening to the class,

Read the textbook;

Do examples independently;

After making an inspection, the teacher provide guidance and answer questions to individual students.

Students practice in class.

Students summarize by themselves.

Above steps develop students reading ability, let them clarify the problem-solving idea and teach them how to apply the formula. After that students have their own understanding, which can provide further learning method and model.

When finding the maximum value of function, we should be clear about the following points.

Change $\frac{a+b}{2}\geq\sqrt{ab}$ into $ab \leq \left(\frac{a+b}{2}\right)^2$.

a,b are positive numbers.

a+b is certain.

If and only if “=” is found, we can find the maximum value.

6. Cultivate student's self-conscious exploration spirit.

Today, with the implementation of quality education, teaching can not only impart knowledge, but also cultivate students' innovative ability and exploration spirit, so as to promote the development and improvement of students' comprehensive quality. Therefore, an important task of teaching is to develop students' thinking; The author's idea is that if we teachers correctly handle the following two aspects, it is helpful to cultivate students' self-conscious exploration spirit.

Complex problems should not be simplified unilaterally.

The difficulties in teaching are exactly what students need to think actively. As teachers, we can not simply pursue the effect of turning difficulties into easy ones in teaching, so that students lose the experience and process of thinking, but should create a good problem situation, which can help students obtain the progress and development of thinking in the process of solving difficulties.

Both proof and conjecture process should be taught.

Polya, a famous mathematics educator, pointed out that “Guess first, prove later -- that's the way most discoveries are made.” Conjectures formed by induction, analogy, intuition and association can enrich students' knowledge and improve their creativity. For instance, after the example "a、$b\in R^+$， a≠b, prove that $a^5+b^5 > a^3b^2+a^2b^3$"， teacher asked students what can they think about. Then some students guessed that "a、$b\in R^+$，n、$m\in N$ and $a^n+b^n > a^{n-m}b^m+a^mb^{n-m}$" is founded after observing and analysing the structure of formula and the exponential of letters and gave some methods to prove that.

This is the result of students' thinking sparks and exploration spirit. Teachers should encourage and cultivate this spirit so that students' creative and exploratory spirit can be carried forw

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