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Computational Thinking: The Third Paradigm for Mathematical Modeling Teaching

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Abstract. Mathematical modeling provides a good platform for guiding students to flexibly apply mathematical knowledge to solve practical problems, and universities have also launched rich mathematical modeling teaching activities. However, in the process of university mathematical modeling teaching, the drawbacks of two traditional teaching methods that focus on cases and models have gradually emerged. Therefore, this article suggests using computational thinking as a third paradigm for mathematical modeling teaching. This paper shows the computational thinking in the new teaching. This article also gives some suggestions to nowadays teaching module. The proposal of this concept has a certain reference value for the reform of mathematical modeling teaching.

Keywords: mathematical modeling, computational thinking.

1. Introduction

Mathematical modeling ability is a core competency for students in higher education institutions. Its significance lies not only in its ability to solve practical problems but also in its ability to cultivate students' innovative thinking and problem-solving abilities. However, we found that mathematical modeling teaching in universities is not systematic enough, and the teaching content cannot fully meet the needs of practical problems.

The current teaching model can be broadly divided into two methods: the model & algorithm-centered teaching method and the case-centered teaching method[1]. The basic characteristic of the case-centered teaching method is to start with actual cases, guide students to analyze and study cases, and apply mathematical knowledge to solve practical problems. This teaching method emphasizes student-centered and practicality, and encourages students to acquire knowledge and skills through exploration and practice[2]. However, some universities stick to the book in the teaching process and use specific cases as the main content for written exams, which is not the original intention of mathematical modeling education. What mathematical modeling requires is that students can flexibly apply mathematical tools in completely unfamiliar problem backgrounds, rather than mechanically memorizing existing simple cases.

The teaching method centered on models takes commonly used mathematical tools as the main thread, and focuses on guiding students to learn the basic theory and mathematical principles of algorithms. However, such a teaching way requires a large knowledge base from students and is difficult to learn quickly, requiring teachers to be targeted in the teaching process. In addition, due to the emphasis on mathematical theory, students often show weak model-solving abilities in the learning process.

Therefore, it is necessary to improve the existing teaching model and find a new teaching focus to connect the knowledge system of mathematical modeling. With the introduction of computational thinking as a main thread, mathematical modeling knowledge can be connected by computational thinking as a focus on solving practical problems and communicating mathematical ideas[3]. Moreover, computational thinking can actually serve as the third paradigm of mathematical modeling teaching.

2. Computational Thinking in Mathematical Modeling

The characteristics of computational thinking lie in its abstract way of thinking, which emphasizes the computability of problems. It stresses the decomposition, abstraction, and symbolization of problems, as well as the use of algorithms and mathematical models to solve problems. Computational thinking and mathematical modeling ideas have a high degree of similarity, making it possible to use computational thinking as the third paradigm of mathematical modeling teaching.

2.1 Describe Mathematical Problems via Variables and Relationship

The core of learning mathematical modeling is how to use quantities and relationships to describe mathematical problems[4]. In the process of mathematical modeling, the core work is to translate the problems described in natural language into mathematical language. Its essence is how to use quantities and relationships to describe mathematical problems. Common mathematical quantities include: scalars, vectors, matrices, etc., while relationships exist in various forms such as equation relationships, inequality relationships, functional relationships, etc. In the specific teaching process, it is not only necessary to spread specific algorithms and knowledge, but also to guide students to understand the background of problems and master how to discover mathematical problems from natural language. This process will help students use computational thinking to connect the knowledge they have learned.

2.2 The Mutual Conversion between Discretization and Continuity

Discrete and continuous are two crucial concepts in mathematics. When transitioning from discrete models to continuous models, significant differences arise, but at the same time, for certain problems, there is a strong interconnectedness between them. For instance, in mathematical models related to population prediction, some models abstract the process as a sequence of discrete steps (such as Leslie model), while others utilize differential equations of continuous functions to depict population changes over time. When utilizing computers to approximate solutions to differential equations, a common practice is to discretize the solution space, making it amenable to numerical methods like Euler's method or Runge-Kutta method. This essentially represents the transformation of continuous problems into discrete ones. Similarly, the algorithms behind many machine learning models echo this dialectical union between the discrete and the continuous.

2.3 The Unification of Symbol Solutions and Numerical Solutions

In many conventional problems, deriving and analyzing exact solutions is a crucial task. However, it is also important to recognize that not all mathematical problems have or are suitable for exact solutions, and in such cases, the importance of numerical approximation becomes self-evident. For example, when solving certain partial differential equations with irregular boundary conditions, we can use numerical methods to mesh the solution space and visualize the results. And after obtaining the numerical solution, we can further discuss its stability based on the properties of the symbolic solution. For some large-scale planning problems, some multi-constraint programming problems are not suitable for solving using traditional methods. In this case, it is necessary to consider using some intelligent optimization algorithms (such as genetic algorithms) to solve the problem, but this solution is often a numerical approximation solution. However, for some problems, blindly using intelligent optimization algorithms may not be a good choice. Therefore, the purpose of computational thinking in teaching is to guide students to understand which problems are suitable for exact solutions and which problems are suitable for numerical solutions. When using numerical solutions, how to explore the error of the results and analyze the sensitivity of the model. This is a relatively large concept.

2.4 The Transformation and Analogy Thinking

Transformation thinking is a very valuable idea in the mathematical subject. Students' exercise and cultivation of transformation thinking can help them not only grasp mathematical modeling knowledge but also flexibly transform existing knowledge into complex practical problems[5]. The following takes the 2020B problem of the US High school Mathematical Competition of Modeling (HiMCM) as an example. The problem requires students to use math modeling to explore the economic impact of the "plastic bottle ban" proposed by the US. Some students investigate the impact of this ban on the beverage market and innovatively analogize the ecology's predator model (Lokta-Volterra model). They treat plastic bottle beverages and other types of beverages as populations in a competitive relationship in the market system, describe their competitive relationship using a system of differential equations, and adjust parameters to observe changes in solutions through policy influence. This is a very clever idea, and there are many similar examples. The purpose of computational thinking is to actively guide students to broaden their horizons and boldly transfer seemingly unrelated content through transformation and analogy to understand that the essence of mathematical models lies in mathematical processes rather than application areas.

2.5 The Problem-Oriented Statistical Thinking

In the context of the big data era, the understanding and application of statistical knowledge is a hot topic in mathematical modeling, which students are also very interested in[8]. However, students are currently facing difficulties in abstracting problems from specific data, defining and distinguishing between different problems in statistics and machine learning, and forming a comprehensive understanding of the data mining process. The computational thinking approach aims to guide students to focus on the data and the problem itself, use various analytical methods (data processing, hypothesis testing, machine learning, etc.) to decompose and abstract the problem, and construct a familiar perspective from unfamiliar problems. This approach integrates each aspect of the model into problem-solving, which is exactly what problem-oriented statistical thinking aims to convey.

2.6 Knowledge Transfer in Different Scenarios

Mathematical modeling problems come from real-world scenarios and the resulting models are applied in real-world situations. However, scenarios can be diverse in real-world problems, ranging from practical problems around us to current academic frontier issues, all of which can be real-world scenarios for mathematical modeling. In this process, students need to realize that both the life scenario and academic frontier problems have the same mathematical model essence. In the teaching process of mathematical modeling, it is necessary to highly concise different scenarios into the same simple model, and then gradually develop from the simple model to become complex and diverse[6]. From complexity to simplicity, and from simplicity to complexity, this dialectical relationship is also something that computational thinking needs to convey. By grasping different background problems, students can understand how to abstract mathematical models from unfamiliar scenarios, thus reducing students' fear and trepidation.

3. Suggestions to Mathematical Modeling Teaching

Based on the above reflection of computational thinking in the connotation of mathematical modeling, several suggestions for the teaching process are proposed:

3.1 From “Model-Centering” to “Thinking-Centering”

Traditional mathematical modeling teaching often uses the model as the central thread to connect knowledge points, which can make it difficult for students to quickly absorb the knowledge. However, using thinking as the thread to connect the knowledge system of mathematical modeling is more beneficial for students to grasp the core idea and essence of mathematical modeling, while requiring less knowledge of mathematical fundamentals and fields, making it easier for students to learn quickly. Specifically, by using different ideologies as the teaching content and alternating between different mathematical theories such as differential equations, optimization, statistics, students can experience the same mathematical thinking in different mathematical tools and backgrounds[7].

3.2 From “Learning by Teaching” to “Learning by Doing”

Traditional mathematical modeling teaching tends to focus more on "teaching" rather than "learning", meaning that it overly emphasizes the completeness of knowledge points and neglects students' practice. In the process of students' hands-on practice, teachers often lack observation and guidance, and only provide simple suggestions after the practice. This kind of practice makes it difficult for students to understand the standardization and completeness of the process. Therefore, the teaching of mathematical modeling should shift from "learning by teaching" to "learning by doing". Practical work at each step is more beneficial for students to grasp the core idea of mathematical modeling.

3.3 Using a Thread to Connect Models

Traditional mathematical modeling textbooks often focus on models, introducing many backgrounds and cases but finding it difficult to find a complete thread to connect multiple models together. Some textbooks focus on algorithms, but there are many types of mathematical modeling algorithms, which are also difficult to use a clear outline to reveal the evolution or dependency relationship between different algorithms. Therefore, we suggest using a thread to connect different model cases, algorithms, model exploration, simple models to frontier problems, etc., making the student's learning more systematic.

4. Conclusion

Analysis of the current status and issues of mathematical modeling teaching reveals that computational thinking should not only be regarded as the core concept of mathematical modeling, but also as the main content and thread of teaching.By exploring different teaching methods and ideas, students can take the initiative in the classroom, understand the essence of computational thinking, form a unique knowledge framework, and proficiently use mathematical tools. The construction of this teaching paradigm has certain reference significance for the reform of mathematical modeling courses.

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