

Instructional Design of the Concept of Definite Integral

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Abstract. The concept of definite integral is a core content in the teaching materials of advanced mathematics, and it is also a concept with a long history, wide application, rich mathematical and humanistic thoughts, and strong abstraction. The concept covers the limit thought, differential thought, and integral thought, which has significant mathematical abstraction and refined mathematical language expression. It contains rich ideological and political elements. Under the concept of ideological and political education of the curriculum, this paper will imperceptibly incorporate ideological and political elements into the teaching design of the concept of definite integral, in order to improve students' understanding of the concept of definite integral.

Keywords: Curriculum ideological and political; Advanced mathematics; Instructional design; The concept of definite integral; Geometric meaning.

1. Introduction

In December 2016, Xi Jinping, president of China, stressed at the National Conference on Ideological and Political Work in Colleges and Universities that establish morality and cultivate talents should be the central link, and ideological and political work should be carried out throughout the whole process of education and teaching, so as to achieve full staff education, full process education, and all-round education[1].

Advanced mathematics is widely used in engineering, economy, biology and other fields. It is a compulsory and important professional basic theory course for students of science and engineering, management and other disciplines in colleges and universities. This course takes "Establish morality and cultivate talents" and "Three-wide education" as the starting point and foothold of teaching, organically integrates knowledge teaching, ability cultivation, value shaping and soul shaping, and silently penetrates the ideological and political elements of the course into the teaching process.

The learning of this course not only helps students to learn mathematical knowledge, understand the history and culture of mathematics, establish dialectical materialism and correct values imperceptibly, but also broadens their vision, develops their imagination, and develops their ability to analyze and solve problems, so as to meet the needs of the rapid development of science and technology.

The concept of definite integral is a core content in the teaching materials of advanced mathematics, and it is also a concept with a long history, wide application, rich mathematical and humanistic thoughts, and strong abstraction. The concept covers the limit thought, differential thought, and integral thought, with remarkable mathematical abstraction and refined mathematical language expression. It contains rich ideological and political elements. Integral thought has a long history, Democritus' "Mathematical Atomism", Archimedes' "Exhaustion Method", and China's Liu Hui's "Cyclotomic Method" are the embryonic forms of the idea of integration, and they have used these methods to calculate the area and volume of many geometric bodies[2-8]. Through the understanding of the history of integral development, students can have a preliminary understanding of the concept, methods and ideas of definite integral, and realize that mathematics comes from practice and serves the practical problems in life, so as to improve students' ability to explore, innovate and practice, and cultivate students' good habits of initiative and hard work.

2. Teaching Design of the Concept of Definite Integral

2.1 Analysis of learning situation

This course is mainly for freshmen majoring in science and engineering, management, etc., who have certain basic mathematical knowledge and logical thinking ability, and have strong ability to accept new knowledge.

2.2 Teaching objectives

Knowledge objectives: (1) Pursue the production process of definite integral and understand the development history of definite integral; (2) Understand the concept and geometric meaning of definite integral; (3) Experience the process of mathematical concepts from concrete to abstract.

Ability objectives: (1) Understand the mathematical ideas in the concept of definite integral, and be able to solve practical problems by using the four step method of "segmentation, approximation, summation, and limit" defined by definite integral; (2) By analyzing the common phenomena of things, we can summarize the general laws, so as to improve the students' abstract logical thinking ability.

Emotional goals: (1) Through learning the generation process of the concept of definite integral, realize the formation process of mathematical concepts from "intuitive perception to abstract generalization, and from graphic language to mathematical language", and cultivate students' awareness of the combination of numbers and shapes; (2) The definite integral thought of "breaking up the whole into parts, approximate replacement, collect piece into entirety and infinite approximation" contains rich ideological and political elements, enhances students' cultural self-confidence, sense of mission of the times and social responsibility, and helps students establish dialectical materialism from quantitative change to qualitative change, from approximation to accuracy, and the mutual transformation of opposites and unity, as well as the scientific spirit of down-to-earth and excellence; (3) Through group discussion and the application of the four step method of the definition of definite integral to solve Example 2, students' creativity and teamwork ability are cultivated.

2.3 Teaching focus and difficulty

Teaching focus: Understanding the concept of definite integral.

Teaching difficulties: From solving specific practical problems to the abstraction of the concept of definite integral; The application of the four step method of definite integral definition.

2.4 Teaching methods

In the teaching process, we should practice the teaching concept of "student-centered". The teaching methods include teaching methods, heuristic methods, discussion methods and inquiry methods.

2.5 Preview before class

Complete the following tasks in teams:

Task 1. Understand the development history of integral and explore the development process of integral by consulting materials;

Task 2. Find out Liu Hui's "Cyclotomic Method", and discuss its principle and idea;

Task 3. Preview the content of the concept of definite integral and make a mind map.

2.6 Teaching process

2.6.1 Problem Introduction

Firstly, show the picture of Zhaozhou Bridge with courseware, and introduce relevant knowledge of Zhaozhou Bridge. It is the crystallization of the wisdom of the ancient working people, creating a brand new bridge construction in China situation. And it was selected as the world's earliest open shoulder stone arch bridge by the World Record Association, creating the world's largest.



Fig. 1 Zhaozhou Bridge
(Source: Photographic Network)

Secondly, the heuristic method is used to guide students to observe the construction characteristics of Zhaozhou Bridge and think about how to calculate the cross-sectional area of Zhaozhou Bridge?

Finally, the calculation of the cross sectional area of Zhaozhou Bridge is converted into the calculation of the area of curved trapezoid, which is the following example[9].

Example 1. Let function $y = f(x)$ is nonnegative and continuous on interval $[a, b]$. Calculate the area of the curved trapezoid surrounded by line $x = a$, $x = b$, $y = 0$ and curve $y = f(x)$.

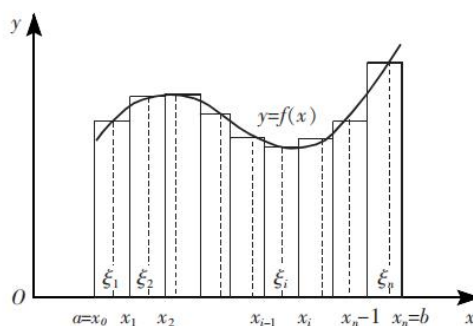


Fig. 2 Curved edge trapezoid

Analysis: Firstly, let the students recall the area formula of rectangular, trapezoidal and other regular figures enclosed by straight lines, and the derivation process of the area formula of circles enclosed by curves, that is, "Cyclotomic Method". Secondly, through the dynamic demonstration of multimedia courseware, students are guided to divide the curved trapezoid into several narrow curved trapezoids by combining the method of "Cyclotomic Method". The curved edge of narrow curved trapezoid can be approximately regarded as a straight line, and the area of narrow rectangle can be approximately replaced by the area of narrow curved trapezoid. Then, the approximate value of the trapezoidal area of the curved edge is calculated, that is, the sum of the areas of these several narrow rectangles. Finally, let the width of the narrow rectangle approach zero to get the exact value of the trapezoidal area with curved edges.

In the whole analysis process, let students intuitively feel the limit thought of "replacing curves with straight lines" and "infinite approximation". Experience the method process of "segmentation (breaking up the whole into parts), approximation (replacing curves with straight lines), summation (collect piece into entirety), and limit (precision)", let students try to explain the principle of each step, and cultivate students' excellent quality of exploration, innovation, and problem solving. This idea of "breaking up the whole into parts, collect piece into entirety" and "replacing the curves with the straight, from the curves to the straight" perfectly reflects the negation law of negation. In addition, this is similar to the ancient Chinese thinker Xunzi's idea of "no accumulation of steps, can't lead to thousand miles".

Example 2. Let an object move in a straight line with known speed, function $v = v(t)$ is continuous on the time interval $[T_1, T_2]$, and $v(t) \geq 0$, calculate the distance traveled by the object during this period s .

Analysis: First, recall the distance calculation formula of uniform linear motion. Secondly, heuristic teaching is used to let students take a bike or ride a bicycle in real life. The shorter the time change is, the smaller the speed change is. Therefore, the speed in a short time can be regarded as a uniform speed. Finally, discuss in groups. In combination with Example 1, solve the problem in four steps of "segmentation, approximation, summation and limit".

2.6.2 Summarize and form concepts

Firstly, guide students to summarize the different points (different backgrounds, geometric problems and physical problems respectively) and the same points (the same method, the same mathematical form) of the above problems, and refine and summarize the concept of definite integral. Secondly, guide students to understand the four step thinking method of "segmentation, approximation, summation and limit" through the expression of definite integral definition. Finally, guide students to understand the concept of definite integral from the perspective of "shape", and clarify the relationship between the concept and geometric meaning.

2.6.3 Thinking and exploration

Complete thinking and exploration in groups.

How to calculate the area of this special figure of China map?



Fig. 3 Map of China
(Source: Baidu)

3. Reveal essence and sublimate problems

(1) The essence of definite integral: limits of sums of special product.

(2) The concept of definite integral reflects the dialectical relationship from macro to micro, and then from micro to macro. Each step contains rich mathematical ideas, ideological and political elements and life philosophy[10-15].

Step 1: Segmentation, that is, breaking up the whole into parts. To guide students not to be afraid when they encounter difficulties in real life, they can be broken down into several small problems that are easy to solve and achieve, and move forward in a down-to-earth manner, thus helping to cultivate students' spirit of striving for progress and progress.

Step 2: Take approximation, that is, replace curve with straight line. Guide students to use known knowledge to overcome unknown problems, so as to get twice the result with half the effort. Realize that both sides of the contradiction of the unity of opposites can transform each other under certain conditions.

Step 3: Sum, that is, collect piece into entirety. This is the same as "no river or sea can be built without small flows, and no one can reach a thousand miles without small steps". It warns students that if they work hard and persevere in learning knowledge or everything in life, they can achieve their life ideals and goals.

Step 4: Take the limit, that is, infinite approximation. Guide students to experience the dialectical view of the unity of opposites of "finiteness and infinity", and the philosophy from quantitative change to qualitative change. It admonishes students to "do not do evil on a small scale, and do not do good on a small scale", positively guides and inspires students, and promotes their positive and healthy growth.

The concept of definite integral is the epitome of life, from which the philosophy of "seeking progress from retreat", "from quantitative change to qualitative change, from finite to infinite, and from approximation to precision" is extracted, which provides a theoretical basis for life. Guide students to make a little progress every day. As long as they accumulate over time, they will surely achieve fruitful results and realize their dreams in a semester or several years.

4. Teaching evaluation

In the teaching design of this lesson, the introduction of Zhaozhou Bridge can not only enhance students' curiosity about the content to be learned, but also stimulate students' patriotic feelings, so that students can feel mathematicians' constant pursuit of knowledge and great achievements. In the process of teaching content, taking mathematical knowledge as the carrier, while introducing mathematical knowledge, combining the rich mathematical history and mathematical culture in the development of mathematics with the scientific value, application value and humanistic value of mathematics, the ideological and political elements of the curriculum run through the teaching process silently. So as to optimize the teaching design, achieve the purpose of ideological education, and cultivate the spiritual world of students imperceptibly.

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